

Development of Mathematical Problem-Solving Ability Using Polya's Problem-Solving Process Combined with Bar Models for Grade 4 Students

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ABSTRACT

The objectives of this study were (1) to evaluate the efficiency of mathematics problem-solving learning activities using Polya's problem-solving process combined with bar models among grade 4 students at the 75/75 efficiency criterion and (2) to compare the students' mathematics problem-solving abilities after participating in the learning activities against the 75 percent benchmark. The participants were 15 grade 4 students from Ban Dong Daeng Nong Phia Khan School, Thailand with cluster random sampling in the second semester of the 2024 academic year. The research instruments included (1) 12 lesson plans integrating Polya's problem-solving process with bar models and (2) a problem-solving ability test comprising 12 subjective questions. Data analysis involved calculating percentages, means, standard deviations, and applying the Wilcoxon Signed Rank Test as a non-parametric method. The results revealed that (1) the instructional activities achieved an efficiency of 77.49/79.11, thereby meeting the 75/75 benchmark, and (2) students demonstrated significantly higher mathematics problem-solving abilities than the 75% criterion at the 0.05 significance level.

Keywords: Mathematics, Problem-Solving, Polya, Bar Models

INTRODUCTION

Mathematics is a fundamental discipline essential for developing 21st century life skills, promoting rational, systematic, and patterned thinking, creativity, and effective problem-solving. These skills are critical tools for analysis, planning, decision-making, forecasting, and practical application in real-life contexts and other disciplines, such as science and technology. The Basic Education Core Curriculum B.E. 2551 (Revised Edition B.E. 2560) emphasizes the development of mathematical competencies in problem-solving, communication, connection, reasoning, and creative thinking, thereby encouraging learners to engage in analytical thinking, use of technology, and collaborative work to prepare for advanced learning and effectively adapt to socio-economic and technological changes (Office of the Basic Education Commission, 2017).

However, studies have shown that the development of mathematical problem-solving abilities among primary students remains suboptimal and is in decline. This is evident from the Ordinary National Educational Test (O-NET) scores in Mathematics for Grade 6 students in the academic year 2023, which averaged 29.96. Similarly, the scores at Ban Dong Daeng Nong Phia Khan School averaged 36.38, which was below 50% of the total score, thereby indicating a national trend of declining performance in Mathematics (National Institute of Educational Testing Service, 2023). Furthermore, this problem extends into lower secondary education, as reflected in the PISA

(Programme for International Student Assessment) results, which assess students' literacy in reading, mathematics, and science every three years. The 2022 PISA results revealed a decline compared to 2018 and continue to show a downward trend (National PISA Center, Institute for the Promotion of Teaching Science and Technology, 2021). Observations of mathematics instruction at the primary level revealed that many students face challenges when solving word problems, lacking the ability to analyze problems and plan solutions, which is indicative of insufficient reasoning and mathematical process skills. This may be attributed to traditional instruction emphasizing rote learning over analytical thinking, and activities focusing more on outcomes than understanding the thinking process, resulting in students' inability to apply knowledge effectively.

Reports from Ban Dong Daeng Nong Phia Khan School during the 2023 academic year showed that grade 4 students exhibited low achievement in the unit on fractions and required extended learning time, indicating that fractions and word problem-solving remain key obstacles in primary mathematics learning. Therefore, enhancing students' mathematics problem-solving ability is essential for sustainable improvement (Academic Department, Ban Dong Daeng Nong Phia Khan School, 2023).

Given the decline in national test scores and the importance of problem-solving in real-life contexts, this study investigates the effectiveness of integrating Polya's problem-solving process and bar models to enhance grade 4 students' mathematical abilities, improve their attitudes toward mathematics, and demonstrate its relevance in everyday life and across subjects.

LITERATURE REVIEW

Polya's Four-Stage Method is a systematic process that fosters logical reasoning and analytical skills, allowing learners to independently discover solutions and apply them to real-life problems. Manikabasagan (2024) found that Polya's model effectively enhances students' mathematical reasoning, especially for non-routine problems, by promoting structured problem-solving approaches and improving symbolic representation. Phuntsho and Dema (2019) reported that Polya's Problem-Solving Model (PPSM) significantly improved both academic achievement and analytical abilities among primary students, recommending PPSM as a core framework for mathematics instruction.

Bar models is a visual representation method developed in Singapore, use rectangular blocks to depict problem elements, making abstract concepts more tangible and problem-solving more accessible. According to Hofer (2015), bar models help simulate mathematical problems and symbolization, fostering understanding of addition and subtraction relationships. Osman et al. (2018) further supported the use of bar models for enhancing students' comprehension of mathematical problems through clear, sequential visualizations. Yeap et al. (2008) confirmed that bar models translate abstract data into visual models, promoting deep analytical understanding and accurate problem-solving.

METHODOLOGY

Research Objectives

1. To evaluate the efficiency of mathematics problem-solving learning activities using Polya's problem-solving process combined with bar models for grade 4 students at the 75/75 efficiency criterion.
2. To compare the mathematics problem-solving abilities of grade 4 students who participated in the learning activities using Polya's problem-solving process combined with bar models with the 75% criterion.

Research Methodology

Population: The population comprised grade 4 students within the Dong Daeng-Nam Sai-Pa Sang Network Schools, Chaturaphak Phiman District, Roi Et Province, under the Roi Et Primary Educational Service Area Office 1, Thailand. The network included three classrooms from three schools with comparable student proficiency levels: Ban Dong Daeng Nong Phia Khan School (15 students), Rong Kham Witthayanukun School (12 students), and Ban Nam Sai Thiam Khae School (19 students). Total population: 46 students, during the second semester of the 2024 academic year.

Sample: The sample consisted of 15 grade 4 students from Ban Dong Daeng Nong Phia Khan School, Dong Daeng Subdistrict, Chaturaphak Phiman District, Roi Et Province, selected through cluster random sampling during the second semester of the 2024 academic year.

Research Instruments

Mathematics Problem-Solving Lesson Plans: 12 lesson plans (one hour each) were developed integrating Polya's problem-solving process with bar models for Grade 4 students. Their quality was validated by five experts.

The average rating was calculated and compared to Likert's five-point rating scale (Srisa-ard, 2002). The mean ratings for individual lesson plans ranged from 4.71 to 4.85, with an overall mean of 4.78 and a standard deviation (SD) of 0.18, indicating a high level of suitability.

Mathematics Problem-Solving Ability Test: A subjective test comprising 12 questions was developed to measure the students' problem-solving ability using Polya's process with bar models. The content validity was verified by five experts, and item-objective congruence (IOC) values ranged from 0.80 to 1.00 (Tayraukham, 2008). The difficulty index ranged from 0.61 to 0.79, the discrimination index ranged from 0.21 to 0.42 (Whitney & Sabers, 1970), and the reliability coefficient, calculated using Cronbach's alpha, was 0.88.

Data Collection

The development of mathematics problem-solving ability using Polya's problem-solving process combined with bar models for grade 4 students was conducted to evaluate the effectiveness of the designed instructional activities. The finalized research instruments were implemented with the selected sample of 15 grade 4 students from Ban Dong Daeng Nong Phia Khan School during the second semester of the 2024 academic year. The data collection process was carried out as follows:

1. The lesson plans, which had been reviewed and revised based on expert feedback, were piloted with grade 4 students at Ban Nam Sai Thiam Khae School, who were not part of the sample group. Prior to implementation, the students in the sample group were prepared by informing them of the learning objectives of the lesson series. The validated and revised lesson plans were then implemented with the sample group of grade 4 students. Instruction was delivered according to the 12 lesson plans, each lasting one hour. During the lessons, formative assessments were conducted using activity worksheets and teacher observations of students' mathematical process skills and desirable characteristics, which were used to calculate instructional process efficiency (E_1).
2. Upon completion of the 12 hour instructional series, a post-test was administered. The test consisted of 12 subjective items measuring mathematics problem-solving ability and a 30-item achievement test on grade 4 mathematics content.
3. Post-test scores were used to calculate the outcome efficiency (E_2), and both sets of data were analyzed using statistical methods to test the research hypothesis. The effectiveness of the instructional activities was analyzed, along with identification of problems, suggestions, and recommendations for improving instructional approaches.

Data Analysis

The efficiency of the mathematics problem-solving instructional activities using Polya's problem-solving process combined with bar models was evaluated against the 75/75 criterion by calculating the process efficiency (E_1) and outcome efficiency (E_2). Students' mathematics problem-solving abilities were compared with the 75% benchmark using the Wilcoxon Signed Rank Test, a nonparametric statistical method.

RESULTS

Efficiency Analysis of Instructional Activities. The researcher evaluated the efficiency of the instructional activities aimed at developing mathematics problem-solving abilities using Polya's problem-solving process combined with bar models among grade 4 students. The process efficiency (E_1) was calculated based on the students' activity worksheets, observed mathematical process skills, and desirable characteristics scores across 12 lesson plans. The results are summarized in Table 1.

Table 1. Efficiency of Instructional Process (E_1) across 12 Lesson Plans

Plan	Activity Score	Process Skills	Desirable Traits	Total	Mean	SD	E_1 (%)
1	99	171	184	454	30.27	4.28	79.65
2	99	186	171	456	30.40	5.03	80.00
3	100	170	169	439	29.27	3.92	77.02
...	...	...	...	...	...	...	...
12	93	178	187	458	30.53	3.56	80.35
Total	1131	2038	2131	5300	29.44	3.87	77.49

From Table 1, it can be seen that all 12 lesson plans exceeded the 75% benchmark for process efficiency (E_1), indicating their effectiveness in enhancing students' mathematics problem-solving skills. The top three most effective lesson plans were Plan 12 (80.35%), Plan 2 (80.00%), and Plan 1 (79.65%).

In addition to process efficiency, the outcome efficiency (E_2) was calculated based on students' post-instruction achievement scores. Table 2 presents a summary of both E_1 and E_2 values.

Table 2. Process Efficiency (E_1) and Outcome Efficiency (E_2) of Learning Activities.

Component	Max score	Total Score	Mean ($\bar{X}$)	SD	Percentage
Activity worksheets	1440	1131	75.40	12.7	78.54%
Mathematical process skills	2700	2038	135.87	12.88	75.48%
Desirable characteristics	2700	2131	142.07	16.26	78.93%
Total process efficiency (E_1)	6840	5300	353.33	37.96	77.49%
Post-test score (E_2)	450	356	23.73	1.10	79.11%

These results demonstrate that the instructional activities achieved an overall efficiency of 77.49/79.11, surpassing the target benchmark of 75/75.

Comparison of Problem-Solving Ability Against 75% Criterion

The researcher compared students' post-test scores on the mathematics problem-solving ability test, consisting of 12 items, against the 75% benchmark. The test assessed four components: (1) understanding the problem, (2) devising a plan, (3) carrying out the plan, and (4) reviewing the solution. The results were analyzed using the Wilcoxon Signed Rank Test due to the non-normal distribution of the data.

Table 3. Comparison of Problem-Solving Abilities against 75% Benchmark

Component	Max Score	n	Mean	SD	Z	p-value
1. Understanding the problem	24	15	22.60	1.50	3.44	.001*
2. Devising a plan	24	15	20.33	2.55	2.72	.004*
3. Carrying out the plan	24	15	19.67	2.55	2.22	.014*
4. Reviewing the solution	24	15	19.13	1.81	2.13	.017*
Total	96	15	81.73	7.81	2.98	.002*

* Statistically significant at 0.05 level

These findings indicate that the students' problem-solving ability scores, in all four components, were significantly higher than the 75% benchmark at the 0.05 significance level. The highest mean score was in "understanding the problem" ($\bar{X} = 22.60$), followed by "devising a plan" ($\bar{X} = 20.33$), "carrying out the plan" ($\bar{X} = 19.67$), and "reviewing the solution" ($\bar{X} = 19.13$).

DISCUSSION AND CONCLUSION

Efficiency of Instructional Activities Using Polya's Problem-Solving Process and Bar Models The mathematics problem-solving instructional activities integrating Polya's problem-solving process and bar models for grade 4 students achieved an efficiency of 77.49/79.11, which exceeded the target criterion of 75/75. This indicates that the students attained an average process efficiency score of 77.49% based on activity participation, mathematical process skills, and desirable behaviors across the 12 lesson plans. Additionally, students demonstrated an average post-test achievement score of 79.11%. These findings suggest that the learning activities were effective in enhancing the students' problem-solving skills. The structured approach of Polya's process enabled students to think systematically and apply reasoning effectively, aligning with the findings of Espinal and Gelvez (2019), who reported that using Polya's strategy as an instructional model significantly improved mathematical problem-solving skills among grade 3 and 4 students. The use of clear, step-by-step procedures helped students interpret problems accurately and develop confidence when approaching mathematical challenges.

Furthermore, the integration of bar models in the instructional design contributed to the visualization and understanding of mathematical relationships, which is consistent with the findings of Willyarto, Pane, and Chairiyani (2015). They emphasized that bar models enhanced comprehension and symbolization among primary school learners. The activities in this study were based on a rigorous instructional design process that included alignment with the Basic Education Core Curriculum B.E. 2551 (Revised 2017), expert validation, and iterative

revision. The lesson plans also incorporated real-life scenarios that were relatable to students' daily experiences, which encouraged self-directed learning and promoted problem-solving in meaningful contexts. The role of the teacher shifted to that of a facilitator, who guided and supported the learners throughout the process. This aligns with the findings from Xin (2019), who implemented the Singapore Bar Model within the Conceptual Model-Based Problem Solving (CMPS) framework. Xin found that students with mathematics learning difficulties improved their ability to represent word problems symbolically and solve them effectively.

Similarly, Chien I Lee (2017) found that instructional prompts based on Polya's method significantly enhanced elementary students' learning outcomes. The study showed that when learners received appropriate scaffolding, they were better able to solve problems independently. The findings from this study also resonate with research by Erna Yayuk et al. (2024), who demonstrated the effectiveness of bar models in enhancing number sense and solving story problems among grade 3 students in Indonesia.

Development of Mathematics Problem-Solving Ability Compared with 75 Percent Criterion. The findings revealed that the students achieved an average problem-solving score of 81.73, significantly higher than the 75% benchmark at the 0.05 level. This outcome reflects the effectiveness of combining Polya's process with bar models in enhancing problem-solving skills. The learning activities provided opportunities for active participation, with real-life problem scenarios connected to the classroom content. Through guided practice, students learned to analyse problem statements, identify given and unknown information, and explore multiple strategies for solving the problems. This systematic practice not only improved their retention of steps but also developed their reasoning, accuracy, and ability to verify answers independently. These findings confirm that structured practice and real-world relevance can support deeper understanding and long-term skill development. The study aligns with Pradana (2024), who examined Polya's model among grade 5 students and found that learners applied all four stages systematically. Each component of Polya's framework contributed to a coherent cycle of problem-solving. Riyadi et al. (2021) also found that problem-solving abilities increased progressively across grades 3 to 5, although mastery varied depending on problem complexity. Shah et al. (2021) reported that bar model visualization significantly improved addition and subtraction problem-solving abilities, though it showed limited effects on multiplication and division. Morin et al. (2017) similarly concluded that bar models supported learners with mathematics difficulties by improving accuracy and strategic thinking.

The evidence supports the conclusion that grade 4 students who participated in instructional activities using Polya's problem-solving process and bar models demonstrated significantly higher problem-solving abilities than the 75% criterion.

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REFERENCES

- Ban Dong Daeng Nong Phia Khan School. (2023). Academic department of Ban Dong Daeng Nong Phia Khan School, under the Office of Roi Et Primary Educational Service Area 1.
- Chien I Lee. (2017). An appropriate prompts system based on the Polya method for mathematical problem-solving. *EURASIA Journal of Mathematics, Science and Technology Education*, 13(3), 893–910. <https://doi.org/10.12973/eurasia.2017.00649a>
- Chowwalit Chookhampaeng, Chantaraporn Kamha and Sumalee chookhampaeng. (2023). "Problems and Needs Assessment to Learning Management of Computational Thinking of Teachers at the Lower Secondary Level" *Journal of Curriculum and Teaching*. Vol.12 No. 3. Pages 172 -178. <https://doi.org/10.5430/jct.v12n3p172>
- Chowwalit Chookhampaeng, Taratchaporn Chantararat, Sumalee Chookhampaeng. (2024). "The Outcomes of Competency-Based Program about Language for Communication for Early Primary School Students" *Journal of Education and Learning*. Volume 13(5). pp 199-207. <https://doi.org/10.5539/jel.v13n5p199>
- Erna Yayuk et al. (2024). Effectiveness of using a bar model in solving story problems for elementary school students. *Research and Development in Education (RaDEn)*, 4(2), 804–815. <https://doi.org/10.22219/raden.v4i2.35295>
- Espinal, M. L. M., & Gelvez, D. Y. P. (2019). Polya's method as a pedagogical strategy to strengthen the ability to solve mathematical problems with basic operations. *Zona Próxima*, (31), 8–25. <https://doi.org/10.14482/zp.31.372.7>
- Hofer, C. (2015). The introduction of the Singapore bar model in year 1 problem solving: A personal reflection. *The STeP Journal: Student Teacher Perspectives*, 2(2), 107–117. <http://insight.cumbria.ac.uk/id/eprint/4149/>

- Lingga Nico Pradana. (2024). Problem-solving strategy: Mathematical problem-solving model within the Polya framework. *KnE Social Sciences*, 9(6), 728–740. <https://doi.org/10.18502/kss.v9i6.15327>
- Manikabasagan, K. (2024). The effectiveness of using the Polya model in improving mathematical reasoning skills. *Jurnal Pendidikan Sains Dan Matematik Malaysia*, 14(2), 65–79. <https://doi.org/10.37134/jpsmm.vol14.2.6.2024>
- Morin, L. L., Watson, S. M. R., Hester, P., & Raver, S. (2017). The use of a bar model drawing to teach word problem solving to students with mathematics difficulties. *Learning Disability Quarterly*, 40(2), 91–104. <https://www.jstor.org/stable/44280682>
- National Institute of Educational Testing Service. (2023). National educational test report basic (O-NET) [Online]. Retrieved January 20, 2024, from <https://www.niets.or.th>
- National PISA Center, Institute for the Promotion of Teaching Science and Technology. (2021). PISA 2018 assessment results: Reading, mathematics, and science. Retrieved from <http://www.ipst.ac.th>
- Office of the Basic Education Commission. (2017). Learning standards and indicators for mathematics, science and geography in the subject group for social studies, religion and culture (Revised 2017). Bangkok: The Agricultural Co-operative Federation of Thailand.
- Osman, S., Che Yang, C. N. A., Abu, M. S., Ismail, N., Jambari, H., & Kumar, J. A. (2018). Enhancing students' mathematical problem-solving skills through bar model visualisation technique. *International Electronic Journal of Mathematics Education*, 13(3), 273–279. <https://doi.org/10.12973/iejme/3919>
- Phuntsho, U., & Dema, Y. (2019). Examining the effects of using Polya's problem-solving model on mathematical academic achievement and analyzing ability of the fourth-grade students. *AJESS*, 5(2), 1–8. <https://doi.org/10.9734/ajess/2019/v5i230142>
- Riyadi, T., Syarifah, T. J., & Nikmaturohmah, P. (2021). Profile of students' problem-solving skills viewed from Polya's four-steps approach and elementary school students. *European Journal of Educational Research*, 10(4), 1625–1638. <https://doi.org/10.12973/eu-jer.10.4.1625>
- Shah, M. N., Anjum, F., Chand, S., & Tabassum, R. (2021). Enhancing mathematical word problem-solving skills: Using bar model visualization technique. *International Research Journal of Education and Innovation*, 2(3), 119–128. [https://doi.org/10.53575/irjei.v2.03\(21\)11.119-128](https://doi.org/10.53575/irjei.v2.03(21)11.119-128)
- Srisaard, B. (2002). *Introduction to Research*. Bangkok: Suveriyasarn.
- Tayraukham, S. (2008). *Research methodology for humanities and social sciences* (2nd ed.). Kalasin: Prasan Kanphim.
- Whitney, D. R., & Sabers, D. L. (1970). *Improving Essay Examination III: Use of Item Analysis* (Technical Bulletin No. 11). Iowa City: University Evaluation and Examination Service.
- Willyarto, M. N., Pane, M., & Chairiyani, R. (2015). Mathematics learning method of bar modeling for elementary school students. *Advanced Science Letters*, 21(7), 2328–2331. <https://doi.org/10.1166/asl.2015.6266>
- Xin, Y. P. (2019). The effect of a conceptual model-based approach on additive word problem solving of elementary students struggling in mathematics. *ZDM—Mathematics Education*, 51(1), 139–150. <https://doi.org/10.1007/s11858-018-1002-9>
- Yeap Ban Har. (2008). *Problem Solving in the Mathematics Classroom*