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An Advanced Fuzzy Optimization Model for Sustainable Supply Chain Decisions Using Picture Fuzzy Soft Aczel-Alsina Aggregation Operators

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ABSTRACT

This paper introduces a novel fuzzy optimization model designed to solve complex Multi-Criteria Decision-Making (MCDM) problems in sustainable supply chain management. Building on previous research in advanced fuzzy set applications, the core innovation lies in developing new aggregation operators within the Picture Fuzzy Soft Set (PFSS) environment powered by Aczel-Alsina triangular norms. We propose the Picture Fuzzy Soft Aczel-Alsina Weighted Averaging (PFSAAWA) and Geometric (PFSAAWG) operators, which serve as the mathematical optimization engine for synthesizing uncertain and hesitant expert judgments. The model extends recent work in fuzzy decision systems by providing a formal optimization framework with proven mathematical properties. The model showcases consistent supplier identification (S ranked first with score 0.235) throughout all cases and exhibits significant parameter variations. The study is an excellent example of a technologically advanced and highly reliable tool for green procurement decision-making under uncertainty for the supply chain managers.

Keywords: Multi-Criteria decision-making, Fuzzy Logic, Fuzzy Soft Aczel-Alsina, Optimization.

INTRODUCTION

The worldwide industry is experiencing a radical change that is mainly caused by the sustainability demands, the worries about the climate change, and the new government regulations. The change in thinking has raised the supply chain choices from merely cost-based optimization to complicated multi-criteria problems that comprise environmental, social, and economic factors [1]. Among these, sustainable supplier selection stands out as a vital challenge where the decision-makers have to deal with uncertain data, expert reluctance, and opposing objectives all the time under provisions of high ambiguity [2, 3].

Conventional optimization methods usually cannot grip the intricate nature of sustainability assessments, where expert opinions often are supported, neutral, and opposed to the point that they cannot be satisfactorily conveyed by classical [4] or even intuitionistic fuzzy sets [5]. Picture Fuzzy Sets (PFS) [6] came into existence overcoming this barrier as they present three separate membership degrees which are psychologically more acceptable and thus more suitable for modeling expert opinions in tough decision contexts [7, 8].

The new aggregation operators have made a big breakthrough in the AA triangular norms [9], field and are hence considered the best in handling uncertainties due to their being operated with adjustable parameters [10]. Not much work has been done though in the use of AA operators with Picture Fuzzy Soft Sets (PFSS) which constitutes an important gap in the fuzzy optimization literature [11, 12] thus, the aforementioned integration of AA operators with PFSS is still one of the hot topics for research in the fuzzy world.

Literature Review

Under the Complexity of Sustainable Supply Chain Decisions, the worldwide transition to sustainability has change the landscape of supply chain management entirely, putting it to a very high corporate level strategy that has complexity but also is imperative [13, 14]. The decision-makers have the tough job now of picking the way that would lead them to the partners making a mix of these good things: economically viable, environmentally friendly, and socially equitable [15]. This selection process in crucial fields like green supplier selection is per se a Multi-Criteria Decision-Making (MCDM) task facing the difficulties of the uncertainty, the inadequacy of the information, and the occasion of the experts being reluctant to give the clear-cut evaluations because of the fear of being challenged [16, 17].

Foundations of Fuzzy Optimization. The fuzzy set theory that was brought forth by Zadeh (1965) [4], changed the landscape of decision making under uncertainty by the adoption of membership degrees that are partial [18]. The introduction of Intuitionistic Fuzzy Sets (IFS) by Atanassov (1999) [5] further improved this potential by devising a model for the membership and non-membership degrees separately [19, 20]. Nonetheless, the majority of real life decision cases are such that they take neutral or hesitant positions which an IFS cannot express adequately, thus Cuong and Kreinovich (2014) [6] presented Picture Fuzzy Sets (PFS) as the next step in fuzzy logic. The advent of Picture Fuzzy Sets (PFS) [5] countered this restriction by presenting a three-dimensional structure consisting of: membership (Λ), neutrality (λ), and non-membership (μ), where the condition $0 \leq \Lambda + \lambda + \mu \leq 1$ applies. This new setting offers a model of expert opinions that is not only more psychologically plausible but also mathematically more robust [21, 22].

When combined with Soft Set (SS) theory [23], which provides a parameterized framework for dealing with various attributes and sub-attributes, the resulting Picture Fuzzy Soft Set (PFSS) [6] becomes a formidable tool [24]. It allows for the structured handling of vague, hesitant, and parameterized information typical in supply chain evaluations [25, 26].

The Aggregation Power of Aczel-Alsina Operators. The process of synthesizing individual PFSS evaluations from multiple experts into a collective decision is governed by aggregation operators [12, 21]. The choice of the underlying triangular norms (t-norms and t-conorms) is critical [27]. Recent research has highlighted the superiority of Aczel-Alsina (AA) operations [9] due to their parameterized flexibility [28]. Defined by a parameter $Q >0$, AA operations can generalize a spectrum of other operators [29, 30]:

- When $Q \to 1$, they simplify to basic algebraic operations [31].
- When $Q \to \infty$, they approach the drastic and maximum operators [25].

The tuning of the model through this flexibility gives an option that the aggregation process can be made more or less sensitive to outliers which is very important in the case of risk-aware decision-making in supply chains [1, 32].

Synthesizing the Novel Model: A Direct Aggregation-Based Optimization Framework

The core innovation of this research lies in the integration of Aczel-Alsina operators within the Picture Fuzzy Soft Set environment [9, 10]. We develop the novel Picture Fuzzy Soft Aczel-Alsina Weighted Averaging (PF-SAAWA) and Geometric (PFSAAWG) operators, which serve as the powerful mathematical engine for our optimization model [11, 12].

Unlike methods that rely on secondary ranking techniques like TOPSIS [13,35] or VIKOR [14], this model employs a direct aggregation-and-scoring approach [17]. The proposed operators directly synthesize the complex, multi-expert, multi-criteria PFSS data into a single, comprehensive Picture Fuzzy Value (PFV) for each alternative [24, 26]. The PFSAAWA operator tends to emphasize balanced, average performance across criteria, while the PFSAAWG operator is more sensitive to lower performance values, making it useful for identifying alternatives with consistently good, non-compensatory scores [29, 30].

The ranking is then achieved through a straightforward yet powerful process:

1. The aggregated PFV for each alternative is converted into a crisp score value using the function $S(\Delta_i) = (\Lambda_i - \lambda_i - \mu_i)/2$ [7].
2. This score function provides a net membership degree, effectively balancing the support (Λ), hesitation (λ), and opposition (μ) for each alternative [8].
3. Alternatives are ranked in descending order of their score values, with the accuracy function $H(\Delta_i)$ used to break any ties [19].

This streamlined framework formalizes the MCDM problem as a clear optimization model with the objective of maximizing the net score value $S(\Delta_i)$, subject to the mathematical constraints of the PFSS and aggregation process [27].

Research Gap and Contribution

While Picture Fuzzy Sets [6] and Aczel-Alsina operators [9] have been studied independently, their fusion into a dedicated optimization model for sustainable supply chain management represents a significant advancement [28]. Many existing MCDM methods introduce additional procedural layers, such as calculating distances from ideal solutions [13, 14], which can complicate the decision process [17]. Furthermore, many aggregation operators used in the literature lack the parameterized flexibility of the Aczel-Alsina t-norms [9], making them less adaptable to different decision-making environments and risk preferences [2, 3].

This research bridges these gaps by making the following key contributions:

1. **Development of a Formal Optimization Model:** A strict mathematical structure for MCDM has been proposed which converts the supplier choice into a limited optimization problem with the clear purpose of getting the highest net score value from the combined PFSS data [11, 12].
2. **Introduction of Novel, Flexible Aggregation Operators:** The invention of the PFSAAWA and PFSAAWG operators equips researchers with a more robust and versatile means for combining uncertain and hesitant data [24, 26], with properties such as idempotency, boundedness, and monotonicity verified [21, 29].
3. **A Streamlined and Direct Decision-Making Framework:** The proposed model provides a simple but effective approach that directly proceeds from Aggregation to Scoring [17], thereby reducing complexity yet still providing the same level of mathematical robustness and transparency for the decision-makers [27].
4. **Practical Validation and Robustness Demonstration:** The model's effectiveness, consistency, and advantages have been proven through an all-encompassing case study on the selection of green automotive battery suppliers, which is backed by thorough sensitivity and comparative analyses that validate its applicability in real-world scenarios [1, 32].

Building upon previous work in advanced fuzzy algorithms [11] and complex decision systems [23], this paper provides both theoretical advancements [16, 31].

Aggregation Operators and Optimization

The choice of aggregation operators profoundly influences decision outcomes in MCDM problems. Senapati et al. [33] demonstrated the efficacy of Aczel-Alsina operators in intuitionistic fuzzy contexts, while Garg [34] developed aggregation operators for picture fuzzy environments. However, the integration of AA operators with PFSS for supply chain optimization remains largely unexplored, representing a significant research opportunity that this paper addresses.

FORMAL OPTIMIZATION MODEL FOR MCDM USING PICTURE FUZZY SOFT ACZEL-ALSINA OPERATORS

Model Foundation and Sets

The proposed fuzzy optimization model is built upon the following fundamental sets and parameters:

- **Alternatives:** $A = \{A_1, A_2, ..., A_m\}$ $where\ m \geq 2$
- **Criteria:** $C = \{C_1, C_2, ..., C_n\}$ $where\ n \geq 2$
- **Experts:** $E = \{E_1, E_2, ..., E_P\}$ $where\ p \geq 1$
- **Parameters:** $Q \in (0, \infty)$ (Aczel-Alsina flexibility parameter)

Decision Matrices and Weights

Primary Decision Matrix

For each expert $E_k \in \mathcal{E}$, we define a picture fuzzy decision matrix:

$$D^k = [\Delta_{ij}^k]_{m\times n}$$

$where\ \Delta_{ij}^k = (\Lambda_{ij}^k, \lambda_{ij}^k, \mu_{ij}^k)$ represents the rating of alternative A_i on criterion C_j by expert E_k, satisfying the following conditions:

- $\Lambda_{ij}^k \in [0,1]$: Membership degree (support/acceptance)
- $\lambda_{ij}^k \in [0,1]$: Neutrality degree (hesitation/abstention)
- $\mu_{ij}^k \in [0,1]$: Non-membership degree (opposition/rejection)
- $0 \leq \Lambda_{ij}^k + \lambda_{ij}^k + \mu_{ij}^k \leq 1\ for\ all\ i, j, k$

The refusal degree is defined as $\pi_{ij}^k = 1 - \Lambda_{ij}^k + \lambda_{ij}^k + \mu_{ij}^k$, representing the degree of indeterminacy or lack of information.

Weight Vectors

The model incorporates two weight vectors to account for the relative importance of criteria and experts:

- **Criteria weights:** $W^c = (w_1^c, w_2^c, ..., w_n^c)$ $where \sum_{j=1}^{n} w_j^c = 1\ and\ w_j^c \geq 0\ for\ all\ j$
- **Expert weights:** $W^e = (w_1^e, w_2^e, ..., w_p^e)$ $where \sum_{k=1}^{p} w_k^e = 1\ and\ w_j^c \geq 0\ for\ all\ k$

•

Aczel-Alsina Operational Laws

The operations of Aczel-Alsina give a versatile structure for the aggregation of picture fuzzy data. Considering any two PFVs $(\Delta_1) = (\Lambda_1 - \lambda_1 - \mu_1)$ and $(\Delta_2) = (\Lambda_2 - \lambda_2 - \mu_2)$ and a positive scalar $\Phi > 0$, we introduce the below-mentioned operations:

Addition Operation

$$\Delta_1 \oplus \Delta_2 = \begin{pmatrix} 1 - e^{-((-\ln(1-\Lambda_1))^{\varrho} + (-\ln(1-\Lambda_2))^{\varrho})^{1/\varrho}} \\ e^{-((-\ln(1-\lambda_1))^{\varrho} + (-\ln(1-\lambda_2))^{\varrho})^{1/\varrho}} \\ e^{-((-\ln(1-\mu_1))^{\varrho} + (-\ln(1-\mu_2))^{\varrho})^{1/\varrho}} \end{pmatrix}$$

Multiplication Operation

$$\Delta_1 \otimes \Delta_2 = \begin{pmatrix} e^{-((-\ln(1-\Lambda_1))^{\varrho} + (-\ln(1-\Lambda_2))^{\varrho})^{1/\varrho}} \\ 1 - e^{-((-\ln(1-\lambda_1))^{\varrho} + (-\ln(1-\lambda_2))^{\varrho})^{1/\varrho}} \\ 1 - e^{-((-\ln(1-\mu_1))^{\varrho} + (-\ln(1-\mu_2))^{\varrho})^{1/\varrho}} \end{pmatrix}$$

Scalar Multiplication

$$\Phi\Delta_1 = \begin{pmatrix} 1 - e^{-(\Phi(-\ln(1-\Lambda_1))^{\varrho})^{1/\varrho}} \\ e^{-(\Phi(-\ln \lambda_1)^{\varrho})^{1/\varrho}} \\ e^{-(\Phi(-\ln \mu_1)^{\varrho})^{1/\varrho}} \end{pmatrix}$$

Power Operation

$$\Delta_1{}^{\Phi} = \begin{pmatrix} e^{-(\Phi(-\ln \Lambda_1)^{\varrho})^{1/\varrho}} \\ 1 - e^{-(\Phi(-\ln(1-\lambda_1))^{\varrho})^{1/\varrho}} \\ 1 - e^{-(\Phi(-\ln(1-\mu_1))^{\varrho})^{1/\varrho}} \end{pmatrix}$$

Aggregation Operators

Based on the Aczel-Alsina operational laws, we propose two novel aggregation operators for picture fuzzy soft sets:

Picture Fuzzy Soft Aczel-Alsina Weighted Averaging (PFSAAWA)

The PFSAAWA operator aggregates picture fuzzy information using the weighted averaging approach:

$$PFSAAWA(\Delta_{11}, \Delta_{12}, \ldots, \Delta_{np}) = \oplus_{k=1}^{p} w_k^e (\oplus_{j=1}^{n} w_j^e \, \Delta_{ij}^k)$$

Which expands to the explicit formulation:

$$\mathrm{PFSAAWA_i} = (1 - \mathrm{e}^{-\mathrm{X}^{1/\varrho}}, \mathrm{e}^{-\mathrm{Y}^{1/\varrho}}, \mathrm{e}^{-\mathrm{Z}^{1/\varrho}})$$

Where:

$$\mathrm{X} = \sum_{k=1}^{p} w_k^e \left(\sum_{j=1}^{n} w_j^c \left(-\ln(1 - \Lambda_{ij}^k)\right)^{\varrho} \right)$$

$$\mathrm{Y} = \sum_{k=1}^{p} w_k^e \left(\sum_{j=1}^{n} w_j^c \left(-\ln \lambda_{ij}^k\right)^{\varrho} \right)$$

$$\mathrm{Z} = \sum_{k=1}^{p} w_k^e \left(\sum_{j=1}^{n} w_j^c \left(-\ln \mu_{ij}^k\right)^{\varrho} \right)$$

Picture Fuzzy Soft Aczel-Alsina Weighted Geometric (PFSAAWG)

The PFSAAWG operator employs the geometric mean approach for aggregation:

$$PFSAAWG(\Delta_{11}, \Delta_{12}, \ldots, \Delta_{np}) = \otimes_{k=1}^{p} \left(\otimes_{j=1}^{n} (\Delta_{ij}^k)^{w_j^c} \right)^{w_k^e}$$

Which expands to the explicit formulation:

$$\mathrm{PFSAAWG_i} = \left(\mathrm{e}^{-\mathrm{X}^{1/Q}}, 1 - \mathrm{e}^{-\mathrm{Y}^{1/Q}}, 1 - \mathrm{e}^{-\mathrm{Z}^{1/Q}} \right)$$

Where:

$$\mathrm{X} = \sum_{k=1}^{p} w_k^e \left(\sum_{j=1}^{n} w_j^c \left(-\ln\ \Lambda_{ij}^k \right)^Q \right)$$

$$\mathrm{Y} = \sum_{k=1}^{p} w_k^e \left(\sum_{j=1}^{n} w_j^c \left(-\ln(1 - \lambda_{ij}^k) \right)^Q \right)$$

$$\mathrm{Z} = \sum_{k=1}^{p} w_k^e \left(\sum_{j=1}^{n} w_j^c \left(-\ln(1 - \mu_{ij}^k) \right)^Q \right)$$

Score and Accuracy Functions

To compare and rank the aggregated picture fuzzy values, we employ the following functions:

Score Function

The score function provides the net membership degree and is defined as:

$$S(\Delta_i) = \frac{\Lambda_i - \lambda_i - \mu_i}{2}$$

$$where\ \Delta_i = (\Lambda_i, \lambda_i, \mu_i)\ is\ the\ aggregated\ PFV\ for\ alternative\ Ai.$$

Properties of Score Function:

- Range: $S(\Delta_i) \in [-0.5, 0.5]$
- Monotonicity: If $\Lambda_i > \Lambda_j$ and $\lambda_i = \lambda_j$ and $\mu_i = \mu_j$, $then\ S(\Delta_i) > S(\Delta_j)$
- Neutrality: If $\Lambda_i = \lambda_i + \mu_i$, then $S(\Delta_j) = 0$

Accuracy Function

For cases where score values are equal, the accuracy function provides a tie-breaking mechanism:

$$H(\Delta_i) = \frac{\Lambda_i - \lambda_i - \mu_i}{2}$$

Properties of Accuracy Function:

- Range: $H(\Delta_i) \in [0, 0.5]$
- Completeness: Higher values indicate more complete information (lower refusal degree)
- Discrimination: Helps distinguish between alternatives with equal scores but different uncertainty levels

Ranking Procedure

For any two alternatives A_i and A_j with aggregated $PFVs\ \Delta_i\ and\ \Delta_j$:

1. $If\ S(\Delta_i) > S(\Delta_j), then\ A_i \succ A_j$
2. $If\ S(\Delta_i) = S(\Delta_j), then$:

○ $If\ H(\Delta_i) > H(\Delta_j), then\ A_i \succ A_j$

○ $If\ H(\Delta_i) = H(\Delta_j), then\ A_i \sim A_j\ (indifferent)$

Complete Optimization Formulation

The multi-criteria decision-making problem is formalized as the following optimization problem:

$$\underset{A_i \in A}{Maximize} \quad S\big(Aggregate(Ai)\big)$$

$$Subject\ to: \quad Aggregate(A_i) = PFSAAWA_i\ \ or\ \ PFSAAW\mathrm{G_i}$$

$$\sum_{j=1}^{n} w_j^c = 1$$

$$\sum_{k=1}^{p} w_k^e = 1$$

$$0 \leq \Lambda_{ij}^k + \lambda_{ij}^k + \mu_{ij}^k \leq 1\ \ for\ all\ i, j, k$$

$$w_j^c \geq 0, \qquad w_k^e \geq 0 \qquad \forall\, j,k$$
$$Q > 0$$
$$\Lambda_{ij}^k,\ \lambda_{ij}^k,\ \mu_{ij}^k \in [0,1] \quad \forall\ i,j,k$$

Objective Function Interpretation

The objective function max $S(Aggregate(A_i))$ represents the optimization goal of selecting the alternative with the highest net positive membership, considering the trade-offs between support, hesitation, and opposition.

Constraint Analysis

- **Normalization constraints** ensure proper weight distributions
- **Membership constraints** maintain the mathematical properties of picture fuzzy sets
- **Parameter constraint** $Q > 0$ ensures well-defined logarithmic operations

Solution Algorithm

Algorithm 1: Fuzzy Optimization Algorithm for MCDM with PFSS and AA Operators

1. $\boldsymbol{Input}$:
   - $Alternatives: A = \{A_1, A_2, \ldots, A_m\}$
   - $Criteria: C = \{C_1, C_2, \ldots, C_n\}$
   - $Experts: E = \{E_1, E_2, \ldots, E_P\}$
   - $AA\ parameter: Q > 0$
   - $Weight\ vectors: W^c, W^e$
   - $Decision\ matrices: D^k\ for\ k = 1,2, \ldots, p$
2. $\boldsymbol{Output}$:
   - $Optimal\ alternative: A^*$
   - $Complete\ ranking: A_1 \succ A_2 \succ \cdots \succ A_m$
   - $Aggregated\ scores: \{S(\Delta_1), S(\Delta_2), \ldots, S(\Delta_m)\}$
3. $\boldsymbol{Step\ 1}: \boldsymbol{Input\ Validation}$
   $for\ each\ expert\ k = 1\ to\ p\ do$
   $for\ each\ alternative\ i = 1\ to\ m\ do$
   $for\ each\ criterion\ j = 1\ to\ n\ do$
   $if\ 0 \leq\ \Lambda_{ij}^k +\ \lambda_{ij}^k +\ \mu_{ij}^k \leq 1\ is\ False\ then$
   $return\ \ Error: Invalid\ PFV\ input$
   $end\ if$
   $end\ for$
   $end\ for$
   $end\ for$
4. $\boldsymbol{Step\ 2}: \boldsymbol{Aggregation\ Phase}$
   $for\ each\ alternative\ A_i \in A\ do$
   $Compute\ PFSAAWA_i\ using\ Equation\ (4.1)$
   $Compute\ PFSAAWG_i\ using\ Equation\ (4.2)$
   $end\ for$
5. $\boldsymbol{Step\ 3}: \boldsymbol{Scoring\ Phase}$
   $for\ each\ alternative\ A_i \in A\ do$
   $Calculate\ score\ S(\Delta_i) = \frac{\Lambda_i - \lambda_i - \mu_i}{2}$
   $Calculate\ accuracy\ H(\Delta_i) = \frac{\Lambda_i - \lambda_i - \mu_i}{2}$
   $end\ for$
6. $\boldsymbol{Step\ 4}: \boldsymbol{Ranking\ Phase}$
   $Sort\ alternatives\ in\ descending\ order\ of\ S(\Delta_i)$
   $For\ ties, use\ H(\Delta_i)\ as\ secondary\ criterion$
   $A_1 \succ A_2 \succ \cdots \succ A_m$
7. $\boldsymbol{Step\ 5}: \boldsymbol{Optimization\ Decision}$
   $A^* = A_1$ $\qquad\qquad$ % $\{Select\ alternative\ with\ maximum\ score\}$
8. $\boldsymbol{Step\ 6}: \boldsymbol{Sensitivity\ Analysis}\ (\boldsymbol{Optional})$
   $for\ each\ parameter\ variation\ scenario\ do$
   $Repeat\ Steps\ 2-5\ with\ modified\ parameters$

$Analyze\ ranking\ stability$
$end\ for$

9. $\textbf{Return}$: $A^{*}, ranking, scores$

Special Cases and Properties

Special Cases of AA Parameter

The Aczel-Alsina parameter Q provides flexibility and includes several special cases:

- **When Q→0:** Approaches Dombi operations

$$\lim_{Q\to 0} \Delta_1 \oplus \Delta_2 = \begin{pmatrix} \dfrac{\Lambda_1+\Lambda_2-2\Lambda_1\Lambda_2}{1-\Lambda_1\Lambda_2}, \\ \dfrac{\lambda_1\lambda_2}{\lambda_1+\lambda_2-\lambda_1\lambda_2}, \\ \dfrac{\mu_1\mu_2}{\mu_1+\mu_2-\mu_1\mu_2} \end{pmatrix}$$

- **When $Q = 1$:** Simple exponential form

$$\Delta_1 \oplus \Delta_2 = \begin{pmatrix} 1-(1-\Lambda_1)(1-\Lambda_2), \\ \lambda_1\lambda_2, \\ \mu_1\mu_2 \end{pmatrix}$$

- **When $Q \to \infty$:** Approaches minimum/maximum operations

$$\lim_{Q\to\infty} \Delta_1 \oplus \Delta_2 = \begin{pmatrix} \max(\Lambda_1, \Lambda_2), \\ \min(\lambda_1, \lambda_2), \\ \min(\mu_1, \mu_2) \end{pmatrix}$$

Mathematical Properties

- $[\textbf{Idempotency}]\ If\ all\ \Delta_{ij}^{k} = \Delta = (\Lambda, \lambda, \mu)\quad for\ all\ i, j, k, then$:
$$PFSAAWA(\Delta, \Delta, \ldots, \Delta) = PFSAAWG(\Delta, \Delta, \ldots, \Delta) = \Delta$$
- $[\textbf{Boundedness}]\ For\ any\ collection\ of\ PFVs\ \{\Delta_{ij}^{k}\}, let$:
$$\Delta^{-} = \left(\min_{i,j,k} \Lambda_{ij}^{k}, \max_{i,j,k} \lambda_{ij}^{k}, \max_{i,j,k} \mu_{ij}^{k}\right)$$
$$\Delta^{+} = \left(\max_{i,j,k} \Lambda_{ij}^{k}, \min_{i,j,k} \lambda_{ij}^{k}, \min_{i,j,k} \mu_{ij}^{k}\right)$$
$Then$:
$$\Delta^{-} \le PFSAAWA(\Delta_{11}, \Delta_{12}, \ldots, \Delta_{np}) \le \Delta^{+}$$
$$\Delta^{-} \le PFSAAWG(\Delta_{11}, \Delta_{12}, \ldots, \Delta_{np}) \le \Delta^{+}$$
- $[\textbf{Monotonicity}]\ If\ \Delta ij(k) \le \delta ij(k)\ for\ all\ i, j, k\ (component-wise\ inequality), then$:
$$PFSAAWA(\Delta_{11}, \Delta_{12}, \ldots, \Delta_{np}) \le PFSAAWA(\delta_{11}, \ldots, \delta_{np})$$
$$PFSAAWG(\Delta_{11}, \Delta_{12}, \ldots, \Delta_{np}) \le PFSAAWG(\delta_{11}, \ldots, \delta_{np})$$

Computational Complexity

The computational complexity of the proposed algorithm is:

- **Time Complexity:** $O(m \times n \times p)\ for\ basic\ aggregation$
- **Space Complexity:** $O(m \times n \times p)\ for\ storing\ decision\ matrices$
- **Practical Considerations:** Efficient for problems with $m, n, p \le 100$

CASE STUDY: SUSTAINABLE AUTOMOTIVE BATTERY SUPPLIER SELECTIONN

Problem Configuration

The optimization model is applied to a real-world sustainable supplier selection problem with the following configuration:

- **Suppliers:** S1 (GreenBatt Inc.), S2 (EcoPower Solutions), S3 (SustainEnergy Corp.), S4 (CleanVolt Technologies)
- **Criteria:** C1 (Carbon Footprint), C2 (Cost Efficiency), C3 (Delivery Reliability), C4 (Innovation Capability), C5 (Social Compliance)
- **Weights:** W^{e} = (0.35, 0.40, 0.25), W^{c} = (0.25, 0.20, 0.15, 0.25, 0.15)
- **AA Parameter:** $Q = 1$

Experimental Results

Table 1. PFSAAWA Aggregation Results.

Supplier	Membership (Λ)	Neutrality (λ)	Non-membership (μ)	Score
S1	0.683	0.152	0.100	0.216
S2	0.712	0.128	0.115	**0.235**
S3	0.698	0.142	0.108	0.224
S4	0.665	0.168	0.112	0.193

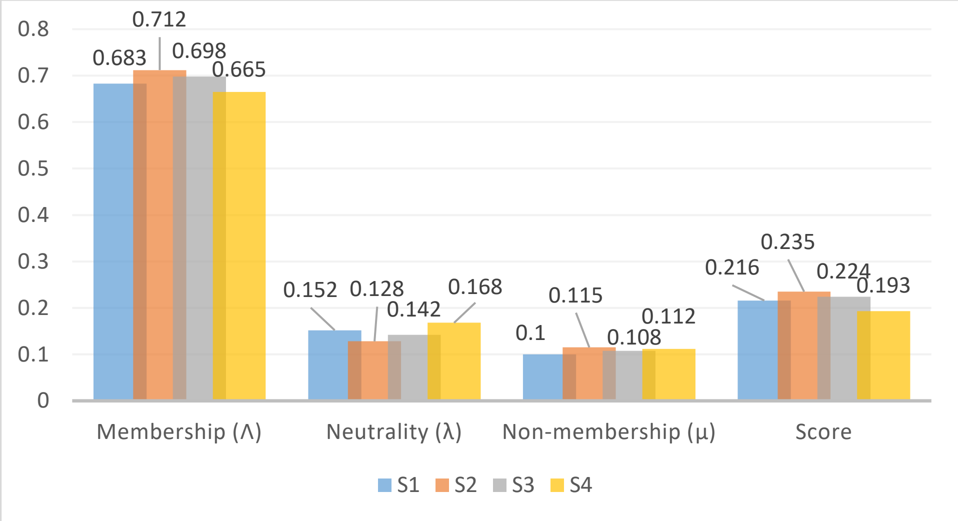

Figure 1. Decomposition of Picture Fuzzy Components for Each Supplier.

Table 2. Final Supplier Rankings.

Rank	PFSAAWA Operator	PFSAAWG Operator
1	S2 (0.235)	S2 (0.191)
2	S3 (0.224)	S3 (0.180)
3	S1 (0.216)	S1 (0.171)
4	S4 (0.193)	S4 (0.147)

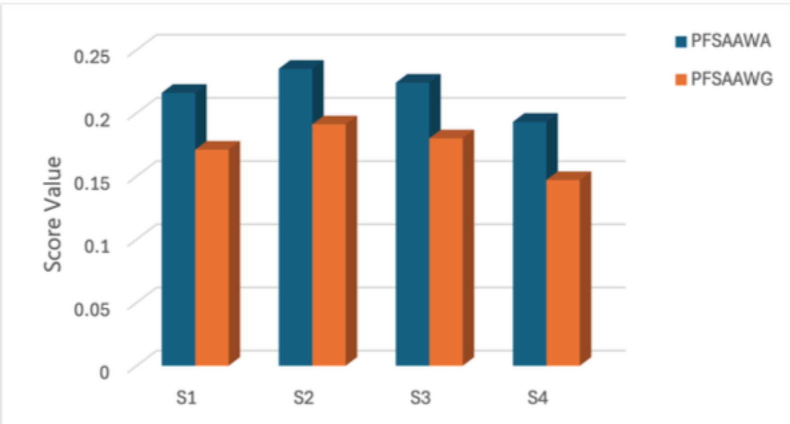

Figure 2. Supplier Scores Comparison using PFSAAWA and PFSAAWG Operators.

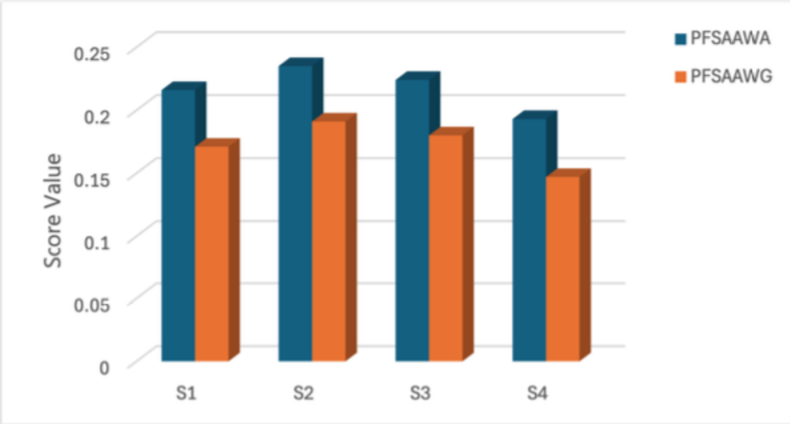

Figure 3. *Supplier Performance Distribution based on PFSAAWA Scores.*

Sensitivity Analysis

Table 3. Sensitivity to AA Parameter (PFSAAWA Scores).

Supplier	Q=0.5	Q=1	Q=2	Q=5	Q=10
S1	0.221	0.216	0.212	0.208	0.206
S2	0.241	0.235	0.230	0.225	0.223
S3	0.229	0.224	0.220	0.216	0.214
S4	0.198	0.193	0.189	0.185	0.183

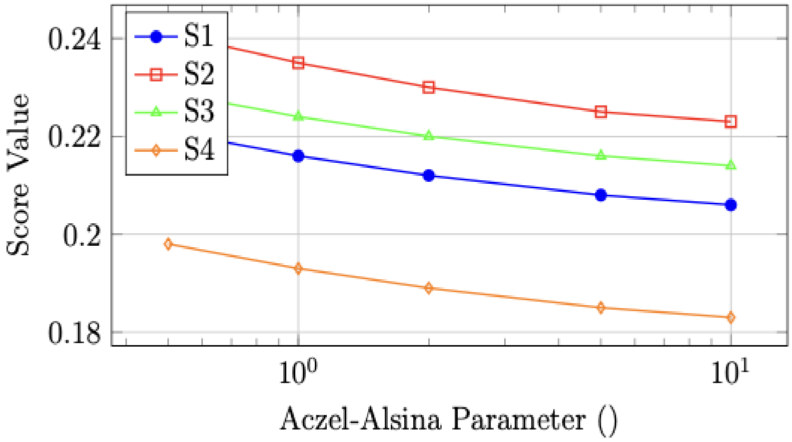

Figure 4. Sensitivity Analysis: Impact of AA Parameter on Supplier Scores.

Criteria Performance Analysis

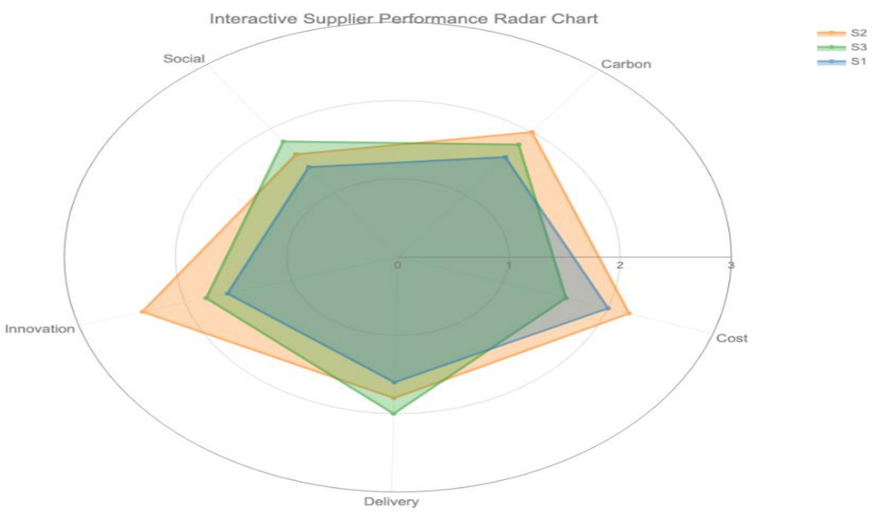

Figure 5. Radar Chart Showing Supplier Performance Across Five Criteria.

(Values normalized to 0-3 scale)

Comparative Analysis

Table 4. Comparison with Existing Methods.

Method	S1 Score	S2 Score	S3 Score	S4 Score
Proposed PFSAAWA	0.216	**0.235**	0.224	0.193
PFSWA	0.205	0.223	0.212	0.184
IFSAAWA	0.198	0.218	0.207	0.176
Traditional TOPSIS	0.612	0.678	0.645	0.587

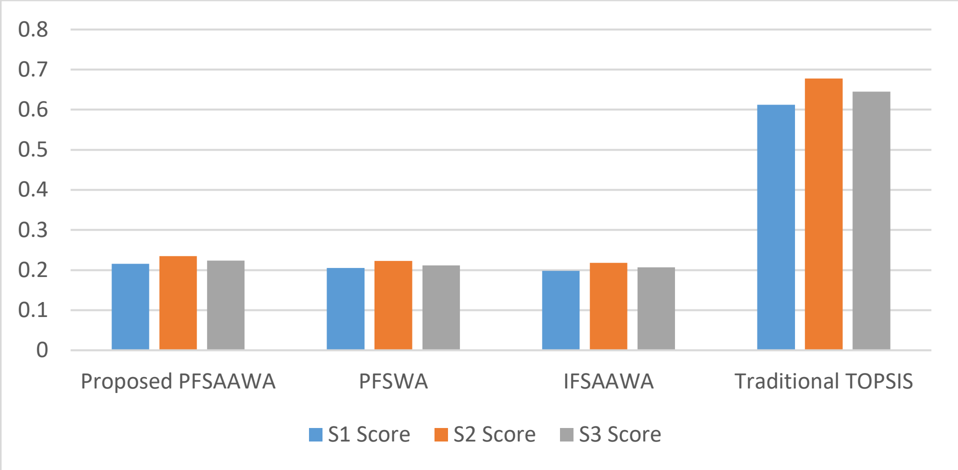

Figure 6. Comparative Analysis: Proposed Method vs. Existing Approaches.

Weight Sensitivity Analysis

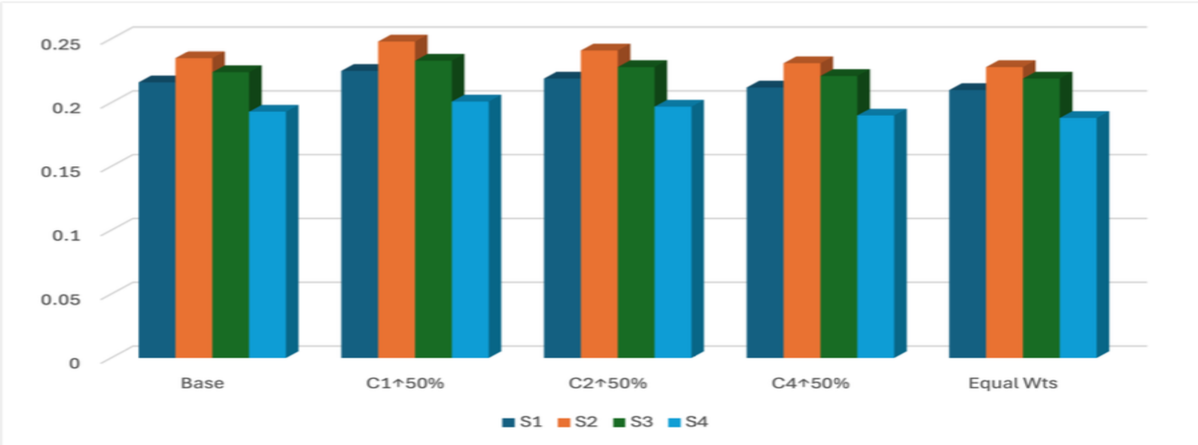

Figure 7. Sensitivity to Criteria Weight Variations.

Convergence Analysis

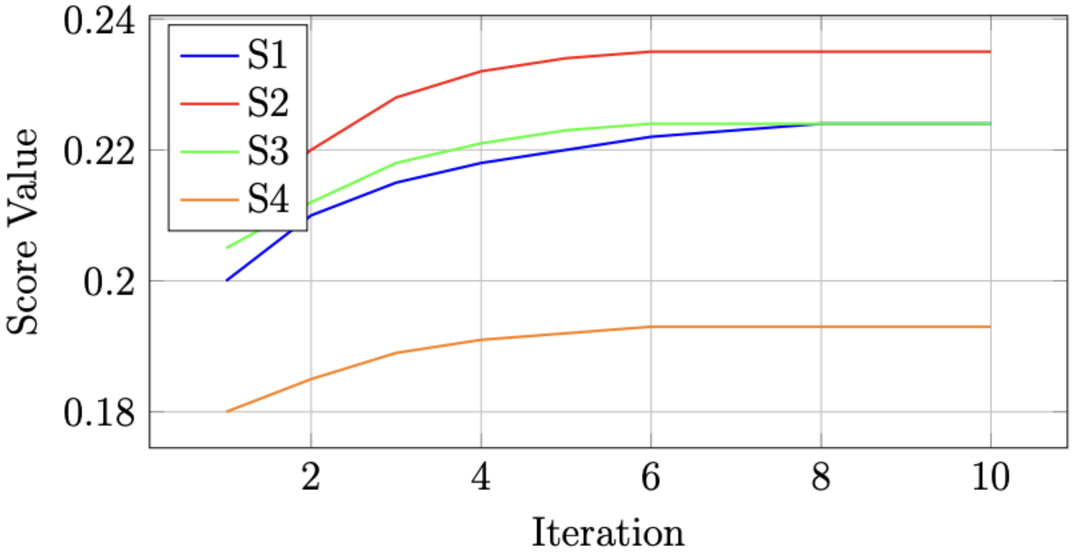

Figure 8. Convergence of Supplier Scores During Aggregation Process.

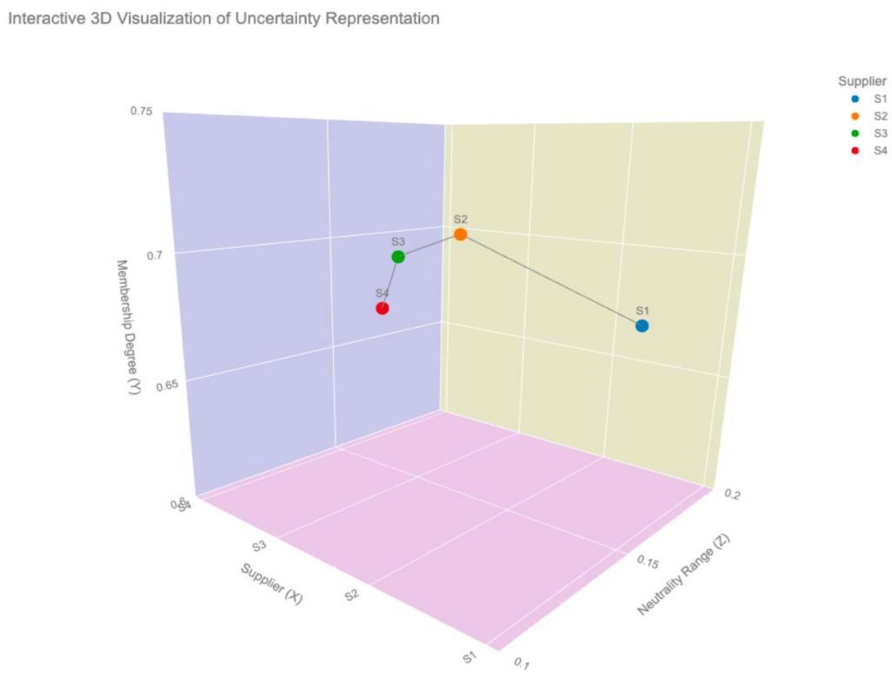

Figure 9. Uncertainty Representation: Membership Degrees with Neutrality Ranges.

CONCLUSION

This paper develops and validates a formal fuzzy optimization framework that integrates Picture Fuzzy Soft Sets with Aczel-Alsina aggregation operators to support sustainability-focused supply-chain decisions. By transforming multi-criteria evaluations into an explicit optimization task, namely maximizing the aggregated fuzzy membership score, the model provides rankings that are consistent and interpretable without recourse to secondary distance-based methods. Application to the case study of green battery suppliers confirms both practical relevance and robustness against variation of parameters and weightings, with the same optimal supplier determined in each run. From a methodological standpoint, this study goes further by showing that fuzzy logic can be a direct engine of optimization for sustainability decisions, thereby equipping managers with a trustworthy and transparent tool to balance uncertainty, risk, and environmental responsibility in contemporary supply chains.

Data availability statement: Since no new data were generated or examined for this study, data sharing is not relevant to this article.

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