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Reconstructing Students' Understanding of Integer Addition and Subtraction: A Didactical Design Grounded in the Theory of Didactical Situations

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ABSTRACT

Mastering integer addition and subtraction is essential for further learning in mathematics. However, students often struggle with persistent conceptual misconceptions. This study goes beyond previous research that has merely documented difficulties or suggested instructional methods without thoroughly examining the underlying barriers. This study presents a didactical sequence based on Brousseau's Theory of Didactical Situations (TDS). It views technology as an extension of cognition and applies this perspective through the IntuMath digital application. Employing a qualitative approach within interpretive–critical paradigms, data were collected through classroom observations, written assessments, and semi-structured interviews. The findings suggest that the proposed design effectively supports students in gradually developing their understanding, transitioning from concrete activities to abstract representations across the four key TDS situations: action, formulation, validation, and institutionalization. Furthermore, using IntuMath expands opportunities for practice and conceptual exploration beyond the classroom setting. TDS helps students progress from real-world experiences with Body-scale Snakes and Ladders (BSL) and Small Snakes and Ladders games to more abstract concepts. Findings show that students were able to construct their own knowledge independently. Additionally, the use of the IntuMath application served as a tool for extended cognition, enabling independent practice and improving conceptual understanding beyond the classroom.

Keywords: Didactical Design, Integer Addition and Subtraction

INTRODUCTION

The foundation of advanced mathematics lies in a strong understanding of numbers. Many students struggle with algebra due to a lack of understanding of number concepts (Boaler & Dweck, 2022). Mastering these concepts is essential, as they are applicable not only in academic contexts but also in many everyday tasks (Barrera-Mora et al., 2018; İPek et al., 2018; Wang & Yang, 2021). Furthermore, these basic operations form the foundation for contemporary cryptography, programming, data representation, and algorithm design (Migallón et al., 2025). A lack of understanding of numbers can lead to serious errors, such as the failure of the Hubble Space Telescope in 1990, which was caused by an arithmetic error in the telescope's programming (Boaler et al., 2022). In the Indonesian mathematics curriculum, number concepts are introduced in 7th grade through various number operations. The most fundamental operations are addition and its inverse, subtraction.

Researchers in mathematics education, particularly in arithmetic, have identified that math anxiety is a significant obstacle to understanding number concepts. Many students face this issue due to an overemphasis on memorizing math facts in the classroom and at home (Boaler & Dweck, 2022; Sidik et al., 2021). Evidence indicates that math anxiety correlates strongly with students' abilities to perform mental arithmetic and make conceptual estimations (Maldonado Moscoso et al., 2020; Si et al., 2016; Szczygieł & Sarı, 2024).

Several contributing factors to math anxiety include cognitive elements, environmental influences, inappropriate teaching methods, individual student traits, and parental attitudes (Kour & Rafaqi, 2024). Boaler & Dweck (2022) notes that students' math anxiety often begins when they face timed testing. The emphasis on memorizing math facts also poses obstacles to understanding number concepts. As students increasingly rely on memorization, they become less motivated to think critically about numbers and their relationships, hindering their numerical understanding (Boaler & Dweck, 2022).

Underachievement in arithmetic is not necessarily due to a lack of understanding of number operations, but rather a lack of flexibility in applying numbers (Boaler & Dweck, 2022). The abstract nature of numbers and their operations is a common source of difficulty for students (D. Permata et al., 2019). Gray & Tall (1994) point out that the key difference between high-achieving and low-achieving students in addition and subtraction lies in using number sense; high-achieving students demonstrate this skill, while low-achieving students often do not. Number sense refers to a person's understanding and intuition about numbers (Dehaene, 2011). Researchers also found that when students worked on arithmetic problems, such as subtraction, the highest-performing students were those who showed the strongest connections between the two sides of the brain: the left brain, which handles factual and technical information, and the right brain, which handles visual and spatial information (Boaler et al., 2022).

Based on the facts above, it can be concluded that many factors hinder students from understanding number concepts, resulting in their failure to understand advanced mathematics. Using Brousseau's framework (Gjone, 2024) as a guide, we may look at these problems through the prism of learning obstacles, broadly classified into three main types: epistemological, ontogenic, and didactic. Epistemological obstacles arise from the limits of concepts or rules that hold only within specific contexts; ontogenic obstacles relate to constraints in students' cognitive development; and didactic obstacles result from how teachers present content and their instructional approaches.

When examined through this framework, the results of the analysis above reflect three learning obstacles according to Brousseau (Gjone, 2024): ontogenic obstacles, as students often lack adequate prerequisite knowledge and experience, which can lead to increased anxiety. Didactic obstacles arise when learning focuses more on procedures than on developing conceptual understanding. Epistemological obstacles, as students struggle with the inherent abstraction of numbers, tend toward incorrect or incomplete reasoning. An alternative instructional design is required to address these identified learning obstacles, offering principled solutions to support the development of students' conceptual understanding.

Previous studies have proposed various designs to enhance students' understanding of integer addition and subtraction. However, these designs often lacked a systematic analysis of students' difficulties. Consequently, the resulting instructional approaches do not provide a strong foundation for meeting learners' needs. For instance, Anton & Abrahamson (2025) developed a body-scale number line and a tabletop number line based on embodied cognition to help students bridge the gap from concrete experiences to abstract representations of integer operations. In a separate study, Cetin (2019) introduced an inversion model as an alternative method for teaching integer operations, utilizing complementary representations of operations and outcomes to reinforce students' understanding of sign rules in both addition and subtraction. More recently, Hastuti et al. (2025) designed a culturally relevant learning model that incorporated traditional dance movements to represent forward and backward motions in integer operations, assisting students in transitioning from informal embodied understanding to formal number line representations. While these studies present innovative design approaches, none were informed by an explicit analysis of learning obstacles, raising concerns about their effectiveness in addressing learners' needs in practice.

Conversely, several investigations focus solely on mapping difficulties without proposing solutions (Zuhriawan et al., 2024), for instance, analyzed the characteristics of students' errors when solving problems and classified them into three categories; Sovia & Herman (2019) identified mathematical difficulties among slow-learning children; and Lewis et al. (2020) reported that students with mathematics learning difficulties often lack conceptual and symbolic understanding of numbers. Collectively, these studies document students' errors and challenges but stop short of offering alternative instructional designs to address the identified problems.

Few studies employ learning obstacle assessments to inform instructional design decisions. However, understanding these obstacles is critical to ensuring that any proposed design adequately meets students' learning needs in real classrooms. As a result, this study suggests an alternative instructional design for integer addition and subtraction that is directly founded in the learning difficulties recognized by the field.

Designing instruction that fosters genuinely meaningful mathematical experiences is essential. Often, knowledge is transferred from the teacher to the students, limiting engagement. In contrast, effective instructional design should enable learners to actively construct their understanding. This approach is most successful when students can interpret and internalize knowledge, allowing them to apply it flexibly and independently beyond specific examples or experiences (Suryadi, 2019). To address this need, Brousseau's (Gjone, 2024) Theory of Didactical Situations (TDS) provides a conceptual framework supporting knowledge construction through four phases of learning: action, formulation, validation, and institutionalization. If these phases are not correctly implemented, students may face obstacles in their learning process.

To optimize the proposed design, we incorporate technological support in a digital application developed by the authors, IntuMath. The application comprises a set of tasks that address integer addition and subtraction and cover integer concepts more comprehensively, thereby affording broader and deeper learning opportunities. The use of IntuMath aligns with the concept of extended cognition (Pritchard, 2018), which emphasizes technology's role in augmenting human cognition by enabling learning that is more efficient, engaging, and unconstrained by space or time. Within the didactical design, technology is thus not treated as an ancillary add-on but as an integrated means of supporting the learning process.

Previous studies that have developed didactical sequences for integer addition and subtraction by explicitly anchoring them in learning challenges and incorporating technology are still scarce. When technology is integrated, it is frequently regarded as an independent component inside the framework known as the didactic tetrahedron (Tall, 1986). Hajizah et al. (2025) created an instructional approach for integer addition and subtraction that addressed students' challenges with mathematical thinking and used technology, using GeoGebra. In that arrangement, GeoGebra mostly let teachers see where integers were on the number line, but students were not involved in building their knowledge. Even without GeoGebra, teaching might continue with a plan that makes students active learners. This does not suggest that technology cannot facilitate knowledge building; in that study, technology was deemed nonessential and appeared predominantly formalistic, aligned with a design conceptualized by the didactic tetrahedron.

The present study makes two significant contributions. First, we develop a didactical design for adding and subtracting integers based on the Theory of Didactical Situations (TDS). This theory prioritizes understanding concepts over merely learning procedures, which makes it well-suited to address the learning challenges we identified. Second, we introduce IntuMath, a digital application that includes various integer activities to provide students with more practice opportunities and enhance their comprehension.

This study tackles deficiencies in previous research that predominantly concentrated on identifying learning impediments without suggesting instructional alternatives or constructing didactic sequences without considering learners' hurdles. Furthermore, research utilizing technology frequently regards it as a separate entity rather than an extension of cognition. We provide a theoretical contribution by creating a TDS-based instructional sequence for integer addition and subtraction, while elucidating the significance of technology in enhancing students' conceptual comprehension. In practical terms, we provide IntuMath as a digital practice space with integer problems that broaden learning opportunities, aid in skill development, enhance conceptual understanding, and assist cognitive growth through integrated learning technologies.

Therefore, creating a pedagogical sequence that addresses recognized learning challenges and integrates technology to enhance students' cognitive processes is essential. The inquiries directing this investigation are: (1) What learning obstacles do students experience in the addition and subtraction of integers? (2) How is the didactical design for integer addition and subtraction developed based on the Theory of Didactical Situations (TDS)? and (3) How is this didactical design implemented in classroom practice?

METHODOLOGY

Research Design

This research employs a qualitative methodology grounded in interpretive and critical frameworks. The interpretive paradigm aims to investigate and thoroughly understand the experiences individuals encounter (Mackiewicz, 2018; Suryadi, 2019). This perspective analyzes the transfer of knowledge from educators to students and examines how learners comprehend that knowledge, recognizing the challenges in this process as obstacles to learning. The critical paradigm focuses on transforming current realities by developing a new educational model informed by these identified challenges.

The interpretive paradigm explicitly investigates how teachers convey knowledge and the methods through which students understand it. Therefore, we conducted classroom observations, administered written assessments, and carried out in-depth interviews. The observations aimed to determine whether the knowledge conveyed was epistemic, facilitating the creation of mathematical concepts, or non-epistemic, potentially hindering learning. The assessments and interviews were designed to evaluate students' understanding of integer concepts. These processes revealed a range of learning obstacles. The critical paradigm aims to enhance the recognized instructional conditions. This study uses learning difficulties as the basis for developing a new didactic design. The design is based on the Theory of Didactical Situations (TDS), which includes four primary situations: action, formulation, validation, and institutionalization. TDS is utilized as it allows students to investigate concepts individually during the action phase. According to Suryadi (2019), an instructional design becomes epistemic when students may independently generate knowledge. The research process is presented in Figure 1.

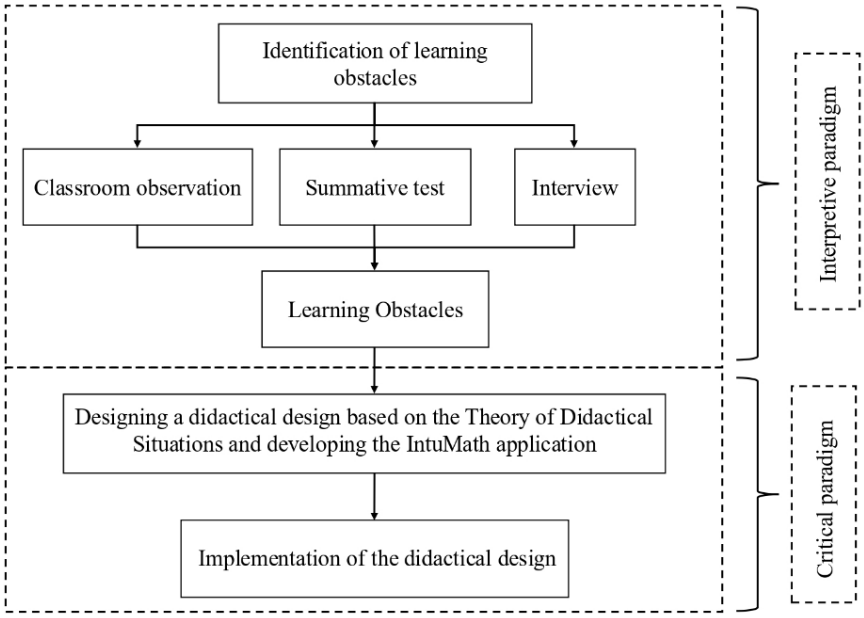


Figure 1. The research flowchart

Participant and Location

The study participants included Grade 7 students from the 2024/2025 academic year, their mathematics teacher, and Grade 7 students from the 2025/2026 academic year at the same school. The research focused specifically on the Grade 7 group from 2024 to 2025, as these students were learning to add and subtract integers. The teacher was included in the study because they directly taught the subject and could provide valuable insights into the teaching methods used.

The didactic design of this study was then applied to the Grade 7 group for 2025/2026, who had not yet learned to add and subtract integers. This group had math skills comparable to those of the previous year's students. This alignment is crucial because the study was tailored to the specific needs and characteristics of the students. Consequently, the implementation aimed to address a group with similar profiles. The learning obstacles and subsequent implementation were analyzed in different academic years to avoid interfering with classroom instruction and to remain aligned with the school's academic calendar.

The research was conducted at a junior high school in Palembang City, South Sumatra Province, Indonesia, for three main reasons. First, the school follows the national curriculum set by the Ministry of Education of Indonesia and uses the official math textbook. This ensures that students' learning aligns with national standards. Second, the student body closely resembles other public schools at the same level, making the findings more applicable and representative. Third, the school supported the research, facilitating participation and ensuring the process ran smoothly.

Data Collection

In qualitative research, the researcher serves as the main instrument and is deeply engaged in data collection. This study collected data using triangulation, employing the following sources.

Observation

Observation involves examining individuals' behaviors within the research environment using unstructured or semi-structured formats. The observer's role can range from non-participant to full participant (Mackiewicz, 2018). In this study, the researcher acted as a non-participant observer, recording mathematics lectures without direct involvement. The observations focused on how the teacher conveyed knowledge and the students' responses to the instructional activities. Subsequently, the field notes were reviewed to identify any learning problems during the lesson.

Test.

In qualitative research, assessments are designed to provide scores and explore participants' conceptual understanding (Mackiewicz, 2018). This study employed five constructed-response questions to evaluate students' grasp of integer addition and subtraction. The assessment was conducted twice. The first test was administered to seventh graders during the 2024/2025 school year to identify their learning obstacles. After implementing a didactic design, the second test was conducted in the 2025/2026 school year to evaluate how well students understood the material following these changes. The interview process was based on the responses from both administrations.

Interviews

According to Creswell & Creswell (2018), interviews are guided conversations that can be either semi-structured or unstructured, with questions that can change based on participants' answers. The study utilized semi-structured interviews with the teacher and a select group of students. Student interviews examined their comprehension and the rationale for their solutions, while teacher interviews investigated the justification for the instructional tactics utilized. Interviews were also used to assess learning problems identified through testing and observation.

Video and Audio Recordings

Audio and video recordings were used to document the entire study process during observations, tests, and interviews conducted both in the classroom and during implementation of the didactical design.

Data Analysis

In qualitative research, data analysis means putting field data in order and structuring it so that it can be easily understood and reported (Creswell & Creswell, 2018). Following (Miles & Huberman, 2014), this study consisted of three fundamental activities: data reduction, data display, and conclusion formulation. The methods employed in this investigation were consistent with this approach.

Data Reduction

Data reduction is the process of selecting, refining, simplifying, abstracting, and transforming raw data into a more understandable form. This process occurs continuously—before, during, and after fieldwork. Before collecting data, reduction involves defining the phenomenon of interest, developing research questions, establishing a conceptual framework, and determining the data collection methods. During the study, data reduction includes activities such as field note-taking and coding. The process continues until the final report is completed. Data reduction is a crucial aspect of analysis and occurs alongside interpretative activity. In this study, data reduction included:: (1) analyzing classroom observations through video recordings; (2) processing students' test responses to evaluate comprehension of integer addition and subtraction; (3) selecting students for interviews and conducting said interviews; (4) transcribing both teacher and student interviews; (5) organizing written test data and interview transcripts; (6) identifying learning impediments; (7) formulating the didactical design; and (8) preparing transcripts of the implementation phase.

Display Data.

Once the data were reduced, they were organized for presentation, which was essential for the conclusion later. In qualitative research, clear and well-organized displays are crucial for helping readers easily understand the results. Data can be shown through narrative prose, infographics, tables, figures, and similar formats. In this study, the displays were aligned with the research questions: (1) learning obstacles were represented through narrative descriptions accompanied by figures; (2) the didactical design was illustrated through narrative with supporting figures; and (3) the implementation results were conveyed through narrative and figures.

Conclusion.

After the data was analyzed and presented, the final step involved synthesizing the findings. These findings correspond to the study's research questions, particularly those related to learning obstacles, the didactical design, and the outcomes of its implementation. Drawing conclusions required verification to ensure the knowledge claims were justified true beliefs. This verification process included member-checking with teachers and students, discussing interpretations among the author team, and seeking insights from experts in relevant fields. Additionally, the findings were validated by thoroughly interpreting all results, considering theoretical frameworks and empirical evidence, especially those from previous studies.

Data Validity

In qualitative research, data trustworthiness is established through four procedures: credibility, transferability, dependability, and confirmability (Lincoln & Guba, 20).

Credibility

Credibility pertains to the degree to which the findings accurately represent reality. Three methods were utilized: First, extended involvement and continuous monitoring. The researchers maintained a close presence during every phase: planning, execution, and evaluation. Then, Triangulation. Three forms of triangulation were employed in our analysis. The triangulation technique assessed the consistency among observations, testing, and interviews. Source triangulation evaluated the alignment between students' test results, interviews with students and teachers, and observations conducted in the classroom. When a learning obstacle arose regarding students' difficulty differentiating operation signs from number signs in integer addition and subtraction, we compared test evidence with observational and interview data to assess cross-source agreement. The application of various theoretical perspectives facilitated a comprehensive interpretation of the findings. The last is consultation with experts. A Focus Group Discussion (FGD) was conducted with subject-matter experts to verify that the proposed didactical design conformed to the principles of the Theory of Didactical Situations (TDS) and remained free of the researchers' personal biases. The focus group discussion offered insights into content validity, the order of activities, and potential challenges before implementation in the classroom.

Transferability

Transferability refers to the degree to which results can be applied in different settings. The research report was crafted with clarity, detail, systematic organization, and comprehensiveness, aligning with the study's objectives and inquiries to facilitate transferability. The findings are presented in various formats, as outlined in the Data Display section, enabling readers to assess relevance and utilize the results as a reference for studies in similar contexts.

Reliability

Dependability refers to the reliability and reproducibility of results when similar methods, contexts, and participant characteristics are applied. This study involved a thorough internal audit of all research procedures conducted by the author team to verify compliance with established protocols, thereby enhancing dependability. Furthermore, validating expert findings guided additional data collection as needed and improved the clarity of the results.

Confirmability

Confirmability is closely aligned with objectivity in quantitative research, ensuring that findings are grounded in participants' experiences and perspectives rather than influenced by researcher bias. To improve confirmability, we preserved all raw data produced throughout the study, encompassing classroom observation recordings, students' written test responses, interview transcripts (from both students and the teacher), and audio recordings of the interviews. We performed transcript confirmation with participants to ensure that the written accounts accurately represented their intended meanings.

RESULTS & DISCUSSION

The findings are divided into three main sections. The first section highlights the learning obstacles identified through classroom observations and a summative test. The second section outlines the instructional design developed to address these obstacles. The third section presents the outcomes of implementing the proposed design.

Learning Obstacles

Students experienced difficulties with integer addition and subtraction, which were identified from two primary sources: (i) classroom instruction observations, and (ii) an interview with the teacher to discuss instructional practices. Additionally, students' written test responses were analyzed. Student interviews were also conducted to gain further insight into their understanding of the topics. Below, we present the results of the classroom observations and the analysis of the test results for integer addition and subtraction.

3.1.1. Classroom Observation

Classroom observations were conducted to identify potential learning obstacles and to examine whether the knowledge presented by the teacher was epistemically aligned with the structure of mathematical knowledge or was non-epistemic, consisting of memorized rules, procedures, or misconceptions.

The observations indicated that the instruction on integer addition and subtraction focused on a simplification procedure that "pairs" the operation sign with the sign of the second number, locally referred to as "dikawinkan." The teacher guided students' attention to both the operation sign and the sign of the second number using the following rules: (1) a plus sign with a plus sign yields a plus; (2) a minus sign with a minus sign yields a plus; (3) a plus sign with a minus sign yields a minus; and (4) a minus sign with a plus sign yields a minus. For example, the expression -5−(-3) was simplified to -5 + 3 (see Figure 2).

However, this simplification introduced some ambiguity because it was unclear whether the resulting sign represented an operation or a number sign. If interpreted as an operation, the expression can be read as "negative five plus three," which produces a clear, definite result. However, if interpreted as a number sign, it could also be read as "negative five plus three," which does not yield a well-defined outcome.

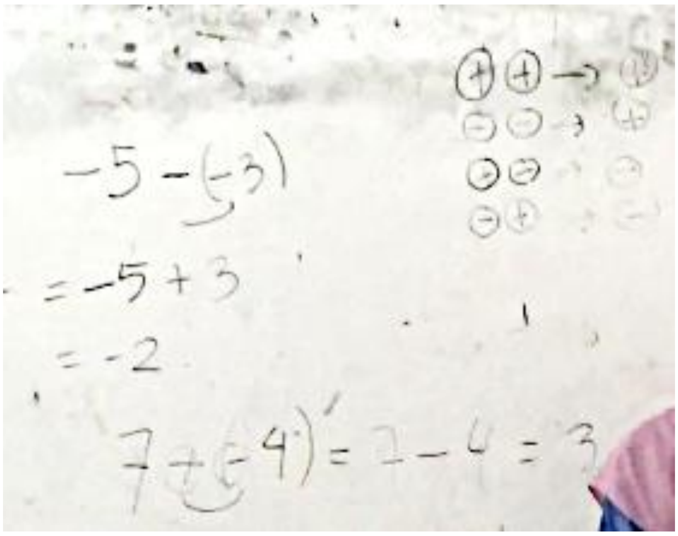


Figure 2. Teacher-led simplification

After simplification, the instructor introduces students to a quick technique for determining solutions (see Figure 3): (1) If both numbers share the same sign, add their absolute values; (2) If the numbers have different signs, subtract their absolute values, using the sign of the number with the greater absolute value. This efficient method demonstrates that the teacher's suggested simplification reveals the sign of the resulting number. However, it presents a serious issue: simplification removes any mathematical operations from the equation, making it unsolvable. Additionally, the teacher makes an error when solving the equation −5 + 3: while the final answer is correct at 2, the method used to arrive at it is flawed.

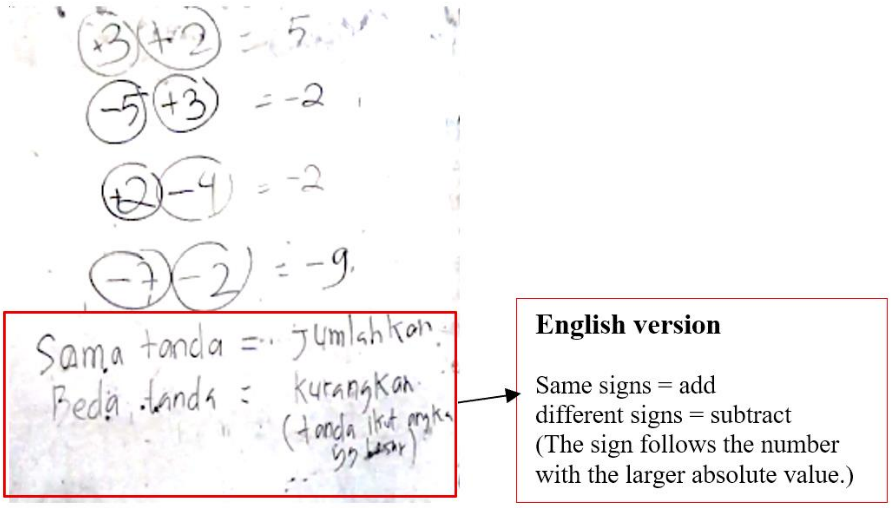


Figure 3. The teachers' expedited technique

The observational data indicate that the teacher's training primarily focuses on procedural approaches, often simplifying symbols without adequately explaining the underlying concepts. This finding is consistent with the work of Fuadiah et al.(2019), which demonstrated that teachers' explanations often lead to misunderstandings due to their inability to distinguish between operational and numerical symbols, thus hindering students' comprehension. Makonye & Fakude (2016) identified that misconceptions surrounding the addition and subtraction of integers arise from insufficient efforts to contextualize the learning process. Another study (Sari et al., 2025) argues that overemphasizing procedural techniques restricts students' ability to develop a deep conceptual understanding.

Effective teaching requires integrating processes, conceptual models, and visual representations to reduce conceptual errors. Hawthorne et al. (2022) emphasize this, who highlight the importance of support and resources for teachers to balance procedural and conceptual methods. Sari et al. (2025) conducted a similar initiative using a visual aid, specifically a mound-hollow model, to improve students' conceptual understanding. However, observations from this study suggest that the teacher's procedural approach may create confusion, as the simplifications fail to clarify whether the symbols should be interpreted as operational or numerical.

The situation is further complicated by the use of a practical 'expedited technique' that can yield correct results in certain instances but does not apply to more complex scenarios. Brousseau (Gjone, 2024) describes this situation as an epistemic block —a learning barrier that arises when knowledge considered accurate in one context leads to misconceptions in another.

Summative Test and Interviews

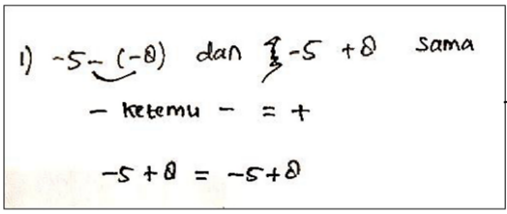
In addition to classroom observations, learning obstacles (LO) were identified through a written assessment designed to evaluate students' understanding of integer addition and subtraction. After the exam, several students were interviewed to confirm their answers and clarify their thought processes. The teacher was also consulted to ensure data triangulation and to explore the rationale for the classroom teaching strategies used.

The assessment tool included five essay questions; however, this paper focuses on two representative items that highlight the predominant patterns of difficulty. The first question asks: "Are the outcomes of $-5 - (-8)$ and $-5 + 8$ identical? Explain your reasoning." Analyzing this question emphasizes not just the final answers provided by students, but also their understanding of the distinction between the subtraction operation and the numerical sign indicating negativity. Additionally, this question explores how students transform the expression $-5 - (-8)$ into $-5 + 8$. The research investigates whether students rely solely on memorized procedures or if they can provide a conceptual explanation, such as using a number line or clarifying the meaning of "subtracting a negative number."

To enhance the analysis, we interviewed students whose responses differed from the majority. Some students, as shown in Figure 4, responded with "minus meets minus yields plus," recalling the procedure taught by their teacher but lacking a meaningful intellectual justification. When further questioned, they could not explain the rationale behind the rule, simply stating that "that is just the quick method." This response indicates that these students only have procedural knowledge and do not understand the significance of integer operations.

According to the Theory of Didactical Situations (Gjone, 2024), this phenomenon represents an epistemological barrier, as an educational approach focused solely on procedural techniques leads students to rely on rote memorization rather than achieving conceptual understanding. These findings are consistent with those of Alyssa P. Lawson et al. (2021), who observed that students often depend on procedural rules for problem-solving. Similarly, Faulkner et al. (2023) noted that students frequently fail to provide logical justifications or to grasp the conceptual reasoning behind their written responses.

To better understand the background of the instructional practices, an interview was conducted with the teacher. The teacher explained that the rationale for teaching procedural rules, such as "minus meets minus yields plus," is based on several considerations. First, this concise rule is an efficient and practical way to help students arrive at correct answers relatively quickly. Second, this technique aligns with traditional math teaching methods, as instructors often prefer to use the same approaches they experienced as students. Third, the teacher believes that students at this level lack substantial experience with abstract thinking, so she opts for practical strategies like procedural rules to aid their comprehension of the material.



English version

$-5 - (-8)$ and $-5 + 8$ are the same
'− meets −' = +
$-5 + 8 = -5 + 8$

Figure 4. Students' response to problem 1

The second difficulty is illustrated in Figure 5. This task aimed to evaluate how effectively students could translate a real-world issue into a mathematical problem, essentially converting a real-world scenario into a mathematical model. This process involves identifying critical information from the task context, relating it to the appropriate numbers or symbols, constructing a mathematical representation of the situation, and finally interpreting the outcome back into its original meaning.

The student's response to Problem 2 reveals a lack of proficiency in translating contextual situations into mathematical representations (see Figure 6). This response was further examined during a student interview. In component (a), the student identified the initial depth as 200 meters without using a negative sign, failing to express the information as "200 meters below sea level." This indicates that the student did not successfully connect the context of ocean depth with the relevant mathematical notation.

Sebuah kapal selam berada di kedalaman 200 meter di bawah permukaan laut. Untuk menghindari daerah dengan tekanan tinggi di dasar laut kapal selam naik 75 meter. beberapa menit kemudian turun lagi 50 meter untuk mencapai lokasi yang ditentukan.
Pertanyaan:
a. **Berada di kedalaman berapa kapal selam setelah naik 75 meter?**
b. **Berada di kedalaman berapa kapal selam setelah turun 50 meter dari posisi tersebut?**
c. **Berapa total jarak yang ditempuh kapal selam?**

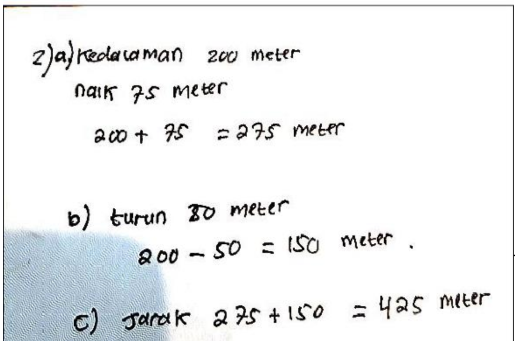
English version

A submarine is 200 meters below the surface of the water. The submarine rises 75 meters to stay away from a high-pressure area near the seafloor. A few minutes later, it goes down another 50 meters to get to the right place.
Questions:
a. How deep is the submarine once it has gone up 75 meters?
b. How deep is the submarine once it has gone down 50 meters from that point?
c. How far has the submarine gone in total?

Figure 5. Problem 2

In section (b), the student recorded the operation 200 − 50 = 150. This indicates that they did not use the previous computation as a reference and instead returned to the initial value. This observation points out a discrepancy in how they represent sequential positional changes. Additionally, in section (c), the student incorrectly calculated the distance traveled as 275 + 150 = 425. The correct calculation should sum the uphill movement (75) and the downhill movement (50), totaling 125 meters. This mistake suggests that the student has not yet differentiated between depth as a relative position and distance as a positive value. Overall, the students' responses indicate that their mathematical understanding remains limited to basic arithmetic operations, lacking a proper connection between contextual meaning and formal mathematical notation.

The student's response in Figure 7 presents a clear mathematical representation. The student recorded the original depth of 200 meters below sea level as a negative number, -200. In part (a), the ascent of 75 meters is shown by adding a positive value: -200 + 75, which results in a new depth of -125. This means the student is now 125 meters underwater. Then, in part (b), the descent of 50 meters is represented by adding a negative value: -125 - 50, leading to a final depth of -175, or 175 meters below sea level. In part (c), the total distance traveled is calculated by adding the upward and downward movements: 75 meters upwards plus 50 meters downwards, resulting in a total of 125 meters.

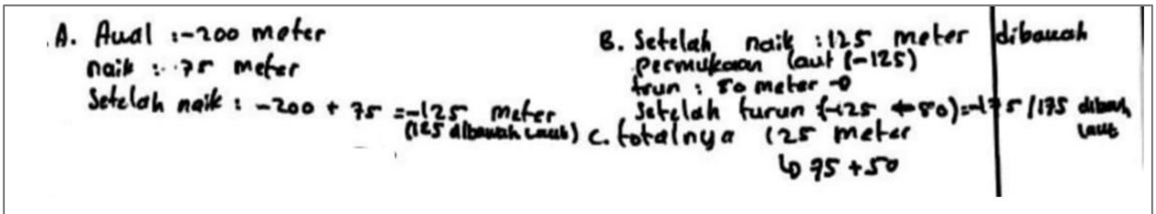


English version

a) depth 200 meters
ascend 75 meters
200 + 75 = 275 meters
b) descend 50 meters
200 − 50 = 150 meters.
c) distance 275 + 150 = 425 meters

Figure 6. Student response to Problem 2

The data suggest that the learning challenges may stem from two primary factors. An educational approach focused solely on procedural steps creates epistemic obstacles for the instructor. Despite previous experience in developing mathematical models to address contextual problems, the students' proficiency in this area has not improved. Along with modeling skills, students need a conceptual understanding of negative numbers and the ability to apply their prior knowledge consistently in multi-step solutions. These limitations indicate the presence of ontogenic obstacles. Brousseau (Gjone, 2024) defines an ontogenic obstacle as a learning difficulty that arises partly from students' insufficient readiness to understand a concept. A student's willingness to learn greatly impacts their ability to understand and integrate new information. Ericsson (2014) emphasizes that students' preparedness and prior knowledge significantly influence the acquisition of new knowledge. Chi et al. (2012) support this by showing that challenges in constructing new ideas often occur when prior knowledge is either underutilized or only partially engaged.



English version

A	Initial: −200 meters Ascend: 75 meters After ascending: −200 + 75 = −125 meters (125 meters below sea level)	B	After ascending: 125 meters below sea level (−125) Descend: 50 meters After descending: −125 − 50 = −175 meters (175 meters below sea level)
		C	Total distance: 125 + 50 = 175 meters

Figure 7. Student's accurate symbolic representation of Problem

Based on classroom observations, written assessments, and interviews, it appears that students have mostly acquired non-epistemic knowledge. This type of knowledge is grounded in procedural memory rather than a solid conceptual understanding. According to Brousseau's Theory of Didactical Situations, non-epistemic knowledge emerges when students are unable to generate ideas on their own and instead merely replicate the teacher's rules.

The learning obstacles identified through observations, assessments, and interviews informed the development of an instructional sequence to address them. This instructional design aims to engage students in activities that promote contextual research, discourse, and the affirmation of ideas, ensuring that the knowledge they gain is epistemic. Epistemic knowledge stems from students' active construction of understanding through meaningful activities that allow for the discovery, verification, and institutionalization of mathematical concepts.

Didactical Design

The pedagogical framework outlined in this research was developed to address previously identified learning challenges. To overcome epistemological issues, the authors created an educational sequence that enhances students' conceptual understanding. This sequence begins with tangible experiences, is supported by visual representations, and culminates in abstract symbolic notation. This approach allows students to develop ideas independently through active learning rather than relying on rote memorization of methods.

The design is based on Brousseau's Theory of Didactical Situations (TDS), which consists of four primary phases: action, formulation, validation, and institutionalization. Instruction starts by reactivating essential information through straightforward exercises that cover concepts students should have mastered before studying integer addition and subtraction. This initial phase addresses the developmental challenges students face concurrently.

The primary learning activities begin with an action scenario in which students use instructional media called Body-scale Snakes and Ladders (BSL) and Small Snakes and Ladders (SSL). BSL is a large snakes-and-ladders board that allows students to move across numerical squares to solve the problems presented physically. In contrast, SSL is a smaller tabletop version played with tokens. These tokens act as "mini-me" representations, mirroring the students' movements while they engage with the BSL under the same rules. Both forms of media are used to address the designated challenges directly. Figure 8 illustrates the integer snakes-and-ladders.

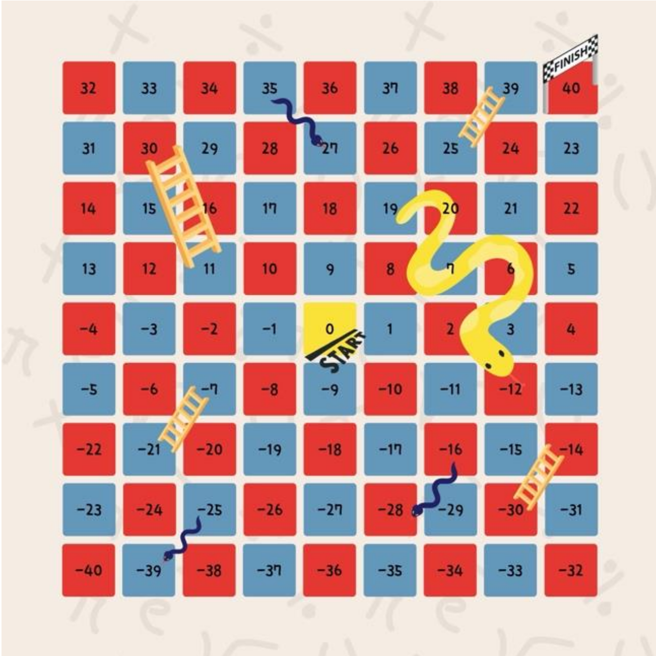


Figure 8. An integer-number Snakes and Ladders board

In the context of teaching integer addition and subtraction, students learn about these fundamental principles. Addition is a binary operation that combines two integers to produce a new number, known as the sum, while subtraction is essentially the addition of the additive inverse.

After engaging in strategies such as Body-Space Learning (BSL) and Structured Space Learning (SSL) during the interactive phase, students are encouraged to identify emerging patterns. For example, they explore the relationship between body orientation and step direction, leading to a practical outcome. They come to understand that addition means moving forward by the amount added, whereas subtraction involves moving backward, which is mathematically similar to adding the negative of a number.

To validate their understanding, students participate in cross-group comparisons, presentations, and peer feedback, all facilitated by the instructor through guiding questions intended to evaluate the strength of their ideas. However, the institutionalization process can present challenges across various settings that may restrict students' ability to apply the knowledge they have developed in new situations.

To address this, the authors designed a pedagogical sequence for the classroom and created a technology-based learning application. This interactive program, called "IntuMath," offers challenges with integers beyond addition and subtraction, helping students develop a deeper understanding. IntuMath can be installed via the following link: bit.ly/IntuMath. It is intended to serve as an external platform where students can practice anytime and anywhere.

The Implementation of Didactical Design

The instruction began with a review of prior knowledge. At this point, the instructor presented simple addition and subtraction problems involving positive numbers—$16 + 8$ and $16 - 8$—and asked the students to solve them. Most students provided accurate answers, although a few required more time to think through the problems. This exercise aimed to reinforce students' prior learning and ensure they were ready to move on to the main learning activities.

The core lesson progressed through the four phases of the Theory of Didactical Situations: action, formulation, validation, and institutionalization. The details of each situation during the initial actions are as follows.

Action.

In this activity, students participated directly in the BSL game, which was designed to enhance their understanding of integer operations. They started at square 0 and picked a number card to determine the second number. The color of the card dictated the operation: blue for addition and red for subtraction. The students' body positions corresponded to the operation sign: they faced right for addition and left for subtraction.

The sign of the second number established the direction of movement: they stepped forward if the number was positive and backward if it was negative, with the number of steps corresponding to the absolute value of the second number. For example, a student at position 0 picks a blue card inscribed with -3. The student turns to the right and then moves backward three squares to -3, representing the equation 0 + (-3) = -3.

In practice, the students showed great excitement, but several made mistakes when dealing with negative numbers. The instructor then provided guidance to help them correct these errors.

After acquiring BSL, the next step was transitioning to SSL, in which students represented their physical actions with tokens on a small number board, serving as a "mini-me" representation. Most students adapted well to this change; however, some struggled to transition from the hands-on experience of BSL to the visual representation in SSL. Their confusion about the direction the token should move—based on the previously learned physical laws—highlighted these challenges.

To help them, the instructor provided additional support in the form of guiding questions related to the token, such as: "If this token is at -1 and the operation is subtraction with -3, in which direction should it move?" and "How many steps does this token need to take?" These prompts helped students focus on the visual representation of the token, allowing them to progress effectively while maintaining consistent rules without reverting to the physical experience in BSL. The students' activities during this stage are illustrated in Figure 9.

The findings indicate that the body plays a crucial role in developing mathematical understanding. Each physical movement, whether advancing or retreating, symbolizes an abstract concept related to integer operations. This aligns with theories of embodied cognition, which propose that cognitive understanding is rooted in sensory experiences and direct interactions with the environment (Lakoff & Johnson, 1999; Wilson, 2002). In this context, body orientation and movement direction go beyond routine physical activities; they act as mechanisms through which students form meaningful concepts of integer operations.

Figure 9. Student activity in the action stage

Subsequent interview findings indicate that students' challenges in transitioning from BSL to SSL stem from several sources, including reliance on BSL and a tendency to revert to prior procedural norms. Dependence on BSL emerged when students, experiencing uncertainty during SSL, returned to the big board to confirm their answers, indicating they had not fully internalized their experience in the small-scale representation. Moreover, several students resorted to procedural norms they had earlier started to forsake. Utilizing BSL, embodied experience liberated them from dependence on mechanical principles like "minus meets minus results in plus". However, during the transition to SSL, several students reverted to these regulations due to a lack of trust in their responses.

Formulation

In formulation, students collaborated in small groups to share their experiences while engaging with BSL and SSL. The discussion outcomes indicated that most groups effectively discerned a basic pattern: body orientation is dictated by the operation sign, while the sign of the second number influences step direction. This discovery was succinct: “body orientation + step direction = operational outcome.”

This formulation process demonstrated that students could integrate their tangible experiences with visual representations while concurrently developing a conceptual comprehension of integer operation principles. Nevertheless, several students had challenges in autonomously recognizing patterns. The challenges were mainly associated with inadequate communication and reflective abilities. Several students had difficulty reviewing their actions, comparing other examples, and articulating them into universally comprehensible generalizations. Consequently, they often halted at procedural replies without effectively developing conceptual norms.

In these instances, the teacher offered scaffolding by asking inquiries such as: “How did you arrive at that solution?”, “Is there an alternative method to solve the problem you encountered?”, and “What instills confidence in the correctness of your answer?” The questions were crafted to challenge students to contemplate their ideas, investigate alternative answers, and enhance their reasoning abilities, directing the formulation process towards the production of conceptual knowledge.

Offering scaffolding at this juncture corresponds with Vygotsky's Zone of Proximal Development (ZPD) hypothesis, which underscores that cognitive advancement transpires via social engagement between learners and more proficient persons (Vygotsky, 1980). Through these encounters, kids receive assistance in addressing challenges they cannot resolve on their own. In this situation, the teacher's prompting questions serve as cognitive support that assists students in connecting real experiences with conceptual knowledge.

Validation

During the validation phase, each group presented the outcomes of their discussions to the class. The responses revealed differences in students' understanding of integer operations. This variation reflects the range of cognitive developmental stages described by Piaget (2011) and Piaget et al., (1973), who argued that the ability to understand abstract concepts evolves gradually from concrete thinking to formal operational thought.

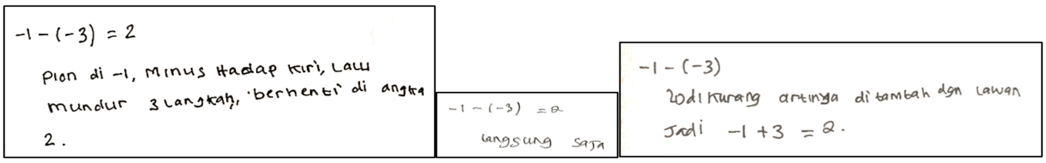
Most groups chose to present their answers in a visual-procedural format. For example, one group explained the operation -1 - (-3) by starting at -1, moving left, and then taking three steps back to arrive at 2. This suggests that some students were still at the stage of using tangible and visual representations, indicating that the validation process was not yet fully effective. The information acquired in the previous phase had not yet been formalized symbolically, and their cognitive processes were still in the concrete operational phase, where mathematical reasoning develops through direct experience and the manipulation of physical objects.

To support these groups, the teacher asked questions directed toward the board or tokens, such as, “If this token moves backward three steps, how would you write the result as an operation?” These inquiries encouraged students to begin linking visual movements with symbolic representations. This process exemplifies assimilation and accommodation, two fundamental concepts in Piaget’s theory, where students adapt their existing cognitive frameworks to understand new representations in formal symbolic form.

Several groups had already achieved symbolic representation. They expressed the equation -1 - (-3) = 2 directly in mathematical notation, without relying on a board or tokens. These groups demonstrated characteristics of formal operational thinking, including the ability to use symbols and engage in abstract reasoning without the need for concrete objects.

In addition, some groups provided explanations based on formal principles and made generalizations, such as stating that "subtracting a negative number is equivalent to adding a positive number." To further challenge these groups, the instructor posed higher-order questions, such as: "Is this rule universally applicable to all subtraction operations involving negative numbers?" and "What is your method of verification?"

These questions affirmed the accuracy of the students' responses and encouraged them to generalize and justify their reasoning at a symbolic level. The students' varied responses are shown in Figure 10.



English version

−1 − (−3) = 2 The token is at −1, faces left (because of the minus), then moves backwards three steps and stops at 2.	−1 − (−3) = 2 Directly obtained	−1 − (−3) Since subtraction means adding the opposite, thus −1 + 3 = 2.

Figure 10. The students’ varied responses

Institutionalization.

In the institutionalization context, the researcher presented problems set in various contexts to evaluate students' ability to solve integer addition and subtraction problems and to assess their understanding of the underlying operational concepts. The assessment consisted of five essay questions, but only two are included in this section, as they are deemed representative of the intended measurement objectives. The problems are shown in Figure 11.

1. Buatlah dua operasi bilangan bulat yang berbeda tetapi menghasilkan hasil akhir yang sama. Tunjukkan perhitunganmu dan jelaskan mengapa kedua operasi tersebut setara.
2. Seorang pendaki melakukan perjalanan di Gunung Bromo yang memiliki ketinggian puncak 2.329 meter di atas permukaan laut. Pendakian dimulai dari pos pertama pada ketinggian 1.800 meter di atas permukaan laut. Dari pos tersebut, pendaki naik sejauh 250 meter untuk mencapai pos pengamatan yang lebih tinggi. Setelah menikmati pemandangan, ia kemudian turun sejauh 400 meter menuju area kawah di lereng gunung.
   Pertanyaan:
   a. Pada ketinggian berapa pendaki berada setelah naik 250 meter dari pos pertama?
   b. Pada ketinggian berapa pendaki berada setelah turun 400 meter dari pos pengamatan?
   c. Berapa total jarak yang ditempuh pendaki dari pos pertama hingga area kawah?

English version

1. Formulate two distinct integer operations that provide the same outcome.
   Present your calculations and explain the rationale for the equivalence of the two procedures.
2. A hiker embarks on an expedition to Mount Bromo, with a summit elevation of 2,329 meters above sea level. The trek starts at the first marker, at an altitude of 1,800 meters above sea level. The hiker ascends 250 meters to get a superior vantage point. After appreciating the vista, the hiker thereafter descends 400 meters into the crater region on the steep side.
   Questions:
   a) What is the hiker's elevation after a 250-meter ascent from the first marker?
   b) What is the hiker's elevation after a descent of 400 meters from the observation post?
   c) What does the hiker traverse the cumulative distance from the first post to the crater region?

Figure 11. The problems in institutionalization

The first problem required students to develop two distinct integer procedures that yield the same outcome and to explain the rationale for their equivalence. The students' responses showcased a range of representations. Some students accurately formulated proper examples, such as $(-5) + 8 = 3$ and $10 - 7 = 3$, and explained that the operations are comparable because both result in 3, despite their different forms. This demonstrates their understanding of flexibility in integer operations and their ability to connect computational results with conceptual thinking.

These findings align with the work of Lamb et al. (2023), who argue that flexibility in integer operations reflects students' capacity to employ various methods or representations that produce equal outcomes. In this case, the students' ability to generate two independent but equivalent operations illustrate their mental understanding of the equivalence between addition and subtraction. The students' responses to this problem are shown in Figure 12.

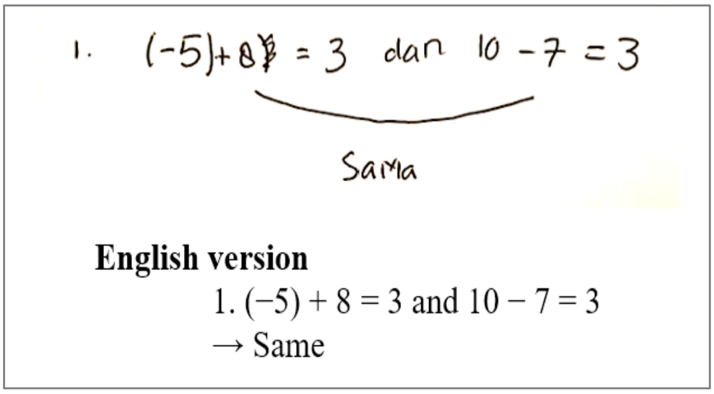
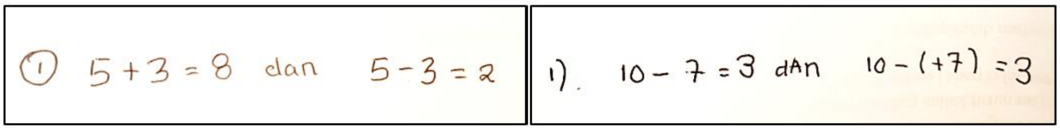


Figure 12. Students' response to problem 1

Some students were found to have performed two separate operations without verifying if the outputs were the same. For example, they wrote $5 + 3 = 8$ and $5 - 3 = 2$. By conducting two separate calculations without checking for equivalence in results, these students demonstrated a lack of understanding of the concept of operational equivalence. Similarly, examples such as $10 - 7 = 3$ and $10 - (+7) = 3$ show that some students repeated the same calculations with minor in changes. While these responses indicate that the students understand the procedure, they do not reflect a grasp of the underlying concepts. Variations in students' responses to the first problem are shown in Figure 13.

These findings align with the research by Ralston & Li (2022) , which indicates that many students still perceive the equals sign as an operational rather than a relational symbol. This is evident in students' answers, which primarily focus on calculating processes rather than on exploring operational equivalence.



English version

5 + 3 = 8 and 5 – 3 = 2

10 – 7 = 3 and 10 – (+7) = 3

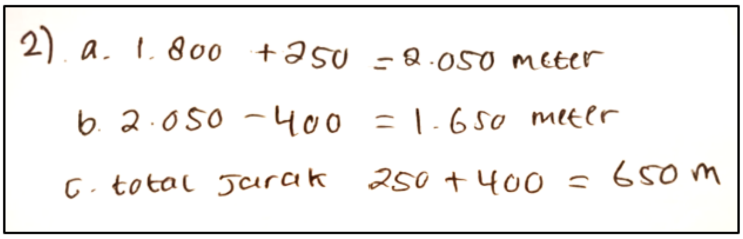
Figure 13. Variations of students' responses to Problem 1

In problem 2, most students accurately identified the starting height as 1,800 meters, the increase of 250 meters, and the decrease of 400 meters in the context of Mount Bromo's elevation. Their mathematical calculations were mostly correct; for example, they correctly calculated 1,800 plus 250 to be 2,050, and then subtracted 400 from 2,050 to get 1,650. This resulted in a total change of 650 meters.

These findings suggest that students who understand the concept of equivalence in integer operations are differentiated from those who are still focused on mechanical, procedure-oriented approaches. This difference is evident when examining their answers to the first problem.

The hiker's final elevation was 1,650 meters above sea level. This indicates that the students were able to take a real-life word problem, simplify it using a mathematical model, and make meaningful connections between the two. Students' responses to problem 2 are illustrated in Figure 14.

The ability to bridge concrete instances with abstract mathematical models, referred to as mathematization, is critical. These results align with those of Jupri & Drijvers (2016) and J. I. Permata et al. (2025), who found that students demonstrated high horizontal mathematization when they could interpret contextual information and construct a mathematical model.



English version

a. 1,800 + 250 = 2,050 meters

b. 2,050 − 400 = 1,650 meters

c. total distance 250 + 400 = 650 m

Figure 14. Student's response to problem 2

Several students continued to make mistakes when formulating the mathematical model. For instance, they expressed calculations such as 1800 - (-250) or 2050 + (-400). While these expressions yield the correct numerical answers, they do not align with the problem's context. This suggests that students are still struggling in differentiate between operational and numerical symbols.

Additionally, many students calculated 1800 - 400 = 1400, neglecting the 250-meter elevation gain. This indicates a difficulty in discerning essential information from the text. These findings align with the study by Agusfianuddin et al. (2020), which found that students struggle to create mathematical models when addressing word problems. They often understand the spoken context but have difficulty representing it symbolically, as evidenced by errors such as writing 1800 - (-250) or 2050 + (-400). Variations in students' responses to the second problem are shown in Figure 15.

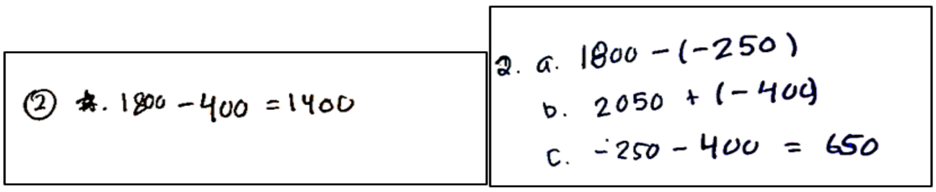
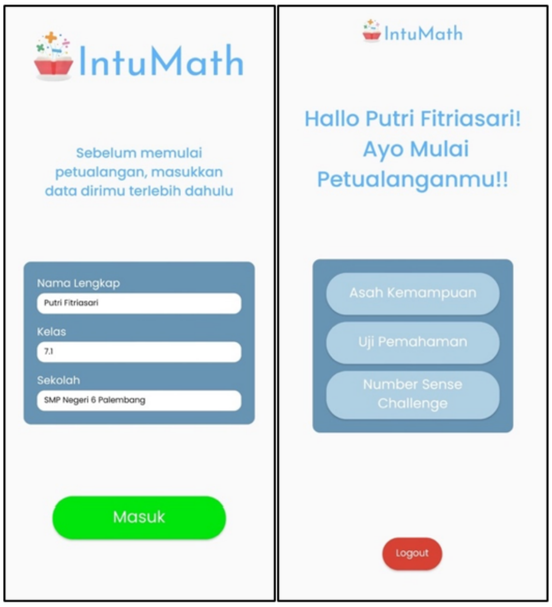


Figure 15. Variations of students' responses to problem 2

After the institutionalization phase, students had the opportunity to improve their understanding of numbers through practice tasks in the IntuMath program. This application is accessible to students at any time and from any location. It offers three types of exercises: (1) Skill Builder, which includes problems categorized by subtopic; (2) Understanding Test, featuring questions that increase in difficulty; and (3) Number Sense Challenge, designed to enhance students' flexible thinking skills. The initial interface of the IntuMath program is shown in Figure 16.

According to the monitoring of student activity in the IntuMath program, all students successfully completed the Skill Builder category. This was achieved because the instructor assigned the task as homework to be completed independently using the app. At the beginning of the class, very few students accessed the Understanding Test or the Number Sense Challenge sections. However, after the instructor informed the class that the final exam questions would be similar to those in IntuMath, nearly every student began preparing in these areas before the session ended.

These findings support previous research that highlights both intrinsic and extrinsic motivation as key drivers of student learning activities. Intrinsic motivators include personal fulfillment, fascination, and curiosity (Bedi, 2023; Qian & Saidin, 2025), while extrinsic motivators encompass grades, incentives, and the expectations of others, such as parents and teachers (Fyfe & Brown, 2020; Siqueira et al., 2020). The observed usage pattern of the IntuMath program suggests that students' engagement in learning is influenced more by external factors than by internal motivation. This underscores the vital role teachers play in fostering students' inherent and ongoing desire to learn.



English version

IntuMath Before starting your adventure, please enter your personal information first. Full Name Class School Enter	Hello, Putri Fitriasari! Let's Begin Your Adventure!! Sharpen Your Skills Test Your Understanding Number Sense Challenge Logout

Figure 16. IntuMath application interface

CONCLUSION

This research highlights the importance of identifying learning obstacles as a foundation for developing instructional techniques that cater to individual needs and characteristics. The findings reveal that students encountered both epistemological and ontogenetic challenges due to an educational focus on procedural methods that overlooked the need for conceptual understanding of integer operations. A didactic design based on the

Theory of Didactical Situations (TDS) was developed, consisting of four key phases: action, formulation, validation, and institutionalization. This design helps students progressively enhance their understanding by transitioning from real-world experiences through Body-scale Snakes and Ladders (BSL) and Small Snakes and Ladders games to more abstract symbolic representations. Implementation results show that students effectively constructed their own knowledge independently. Additionally, the integration of the IntuMath application acted as a tool for extended cognition, expanding students' learning environments beyond the classroom, enabling autonomous practice, and promoting deeper conceptual understanding. This research scientifically advances the creation of mathematics learning designs by analyzing student learning impediments within the TDS framework. The didactic design and the IntuMath application provide a contextual, interactive, and autonomy-enhancing option for mathematics education. Future investigations should implement analogous systems in more mathematical subjects and further examine the function of technology as an extension of cognition in facilitating students' knowledge development.

Conflicts of Interest. The authors declare no conflict of interest.

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Data Availability Statement. The data presented in this study are available on request from the first or corresponding authors.

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