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Advances in Metric Dimension: Variants, Algorithms, and Open Challenges

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ABSTRACT

The metric dimension of a graph is an elementary notion of graph theory measuring the minimum number of reference points (a resolving set) needed to label uniquely all vertices based on their distance to the points. The review of the subject elaborates on the theoretical underpinning of the metric di-mension, its central versions (local, strong, fault-tolerant, and edge metric dimensions), and algorithmic properties, such as NP-hardness and approximations. Besides, we high-light several applications of network verification, robotics, chemistry, biology, and cybersecurity. Open problems and directions for future research are also discussed, noting the growing importance of metric dimension in both theoretical and applied fields.

Keywords: Metric Dimension; Resolving Set; NP-hardness; Robot navigation; Distance-based graph invariants; Approximation algorithms

INTRODUCTION

The metric dimension theory has its origin in the problem of placing points uniquely in a metric space with the fewest number of reference points. In graph theory, the concept was independently formulated by Slater [1,2] and Harary & Melter [3], but the concept has now been developed as a core technique of network structure analysis across many doma ins. Essentially, the metric dimension of a graph is the measurement of minimum number of landmarks so that every vertex of the graph is uniquely determined by its distance vector to the landmarks [4].

Metric dimension is that branch of theoretical and applied mathematics where deep combinatorial ideas are employed in the solution of problems in a broad assortment of applications [5].

From the theoretical perspective, determination of the metric dimension of several classes of graphs (paths, cycles, trees, and more complex networks) discovers intriguing connections with other graph invariants [6]. In the applied context, it has emerged as a workhorse for optimizing real-world systems—such as minimizing the number of sensor placements in navigation networks, constructing fault-tolerant communication protocols, and even computational phylogenetics' analysis of biological data[7]. Mohamed [8] conducted a self-contained introduction to the metric dimension and an overview of several metric dimension results and applications. Amin et al. [9] presented a hybrid approach (WCA_WOA) for computing the metric dimension of graphs that combines the water cycle algorithm and a whale optimization algorithm. Bokhary et al. [10] presented three novel encryption and decryption schemes based on graph theory that aim to improve security and error resistance in communication networks.

Qiang et al. [11] examined several domination notions, such as restrained dominating set (RDS), perfect dominating set (PDS), global restrained dominating set (GRDS), total k-dominating set, and equitable dominating set (EDS) in VGs and introduce their properties through some examples as well. The idea of a minimal intuitionistic dominating vertex subset of an intuitionistic fuzzy graph was presented by Bozhenyuk et al. [12], who also considered the idea of a domination set as an invariant of the intuitionistic fuzzy graph. Jakkepalli et al. [13] proved that the 2-SDM problem is NP-complete for planar graphs and doubly chordal graphs, a subclass of chordal graphs. Certain characteristics of identifying secure dominating sets in the Cartesian product and lexicographic product of two connected graphs were demonstrated by Aquiles et al. [14]. A study of the idea of secure inverse domination in graphs was started by Dagodog et al. [15], who also described the secure inverse dominating set in the join of two connected simple graphs. Total perfect Roman domination is NP-complete for chordal graphs, bipartite graphs, and planar bipartite graphs, as demonstrated by Almulhim et al. [16]. As a novel indicator of graph vulnerability, Antony et al. [17] proposed the idea of paired domination integrity of a graph. The bounds for edge domination number and secure edge domination number for the Mycielski graph of G were established by Gangadharan et al. [18]. Kim et al. [19] obtained closed formulas for domination, secure domination and total domination numbers in middle graphs of rooted product graphs. Dagodog et al. [20] determined the secure inverse domination number of graphs under the binary operations corona and lexico-graphic product. Ortega et al. [21] investigated the super inverse dominating set theory and give the domination number of some special graphs. Akwu et al. [22] defined the upper and lower bounds of perfect locating signed roman domination-function in trees. Diapo et al. [23] initiated an study of disjoint perfect domination theory in graphs and gave some important results. Sigarreta [24] gave bounds or the exact value of the total domination number of new graphs depending on some parameters in the original graph *G*. Mohamed et al. [25] examined the NP-hard problem of finding the minimum secure connected domination metric dimension of graphs. Dayap et al. [26] gave the characterization of the disjoint secure dominating set in the join of two graphs. Merouane et al. [27] proved that the problem of computing the secure domination number is NP-complete for split graphs and bipartite graphs. They also gave bounds comparing the secure domination number with the domination number. Martínez et al. [28] provided a general treatment of the protection of graphs concept, involving the current forms of secure domination and introducing new ones. Banu et al. [29] introduced a new secure distance matrix domination concept by embracing state-of-the-art cryptographic techniques and privacy-protecting measures. Udutohan et al. [30] generalized the study of the concept of disjoint perfect secure domination in graphs. Gomez et al. [31] generalized the concept of fair secure dominating sets by defining the corona of two nontrivial connected graphs and establishing some noteworthy results. Martínez et al. [32] obtained new relationships between the secure total domination number and other graph parameters: namely the independence number, the matching number and other domination parameters. Additional details may be found in the literature [33–42].

The study of **metric dimension** has evolved from a purely theoretical concept in graph theory to a versatile tool with far-reaching applications across multiple disciplines. By determining the minimal number of reference points needed to uniquely identify elements in a network, this framework provides elegant solutions to problems in **network design, robotics, chemistry, biology, and cyber security**.

Preliminaries Notes:

Basic Graph Theory Concepts:

- **Graph (*G*)**: A pair (V, E) where V is a set of vertices and E is a set of edges connecting pairs of vertices [43].
- **Distance ($d(u,v)$)**: The length of the shortest path between vertices u and v in G[44].
- **Connected Graph**: A graph where there exists a path between any two vertices [45].
- **Complete Graph (K_n)**: A graph where every pair of distinct vertices is connected by an edge [46].
- **Path Graph (P_n)**: A graph whose vertices can be listed in order $v_1, v_2, ..., v_n$ with edges between consecutive vertices [47].
- **Cycle Graph (C_n)**: A graph consisting of a single cycle through n vertices [48].

Metric Dimension Fundamentals

- **Resolving Set**: A subset $S \subseteq V$ of vertices in a graph G such that for every pair of distinct vertices $u, v \in V$, there exists at least one vertex $w \in S$ where $d(u,w) \neq d(v,w)$[49].
- **Metric Dimension (dim(*G*))**: The minimum cardinality of a resolving set for G [50].
- **Example**:
- For P_n (path graph), $\dim(P_n) = 1$ (only one endpoint is needed as a landmark).
- For K_n (complete graph), $\dim(K_n) = n$-1 (all but one vertex must be landmarks).

Key Properties

- **Uniqueness**: The distance vector ($d(v,s_1)$, $d(v,s_2)$, ..., $d(v,s_k)$) for any vertex v must be unique with respect to the resolving set $S = \{s_1, s_2, ..., s_k\}$[51].
- **Bounds**:
- For any connected graph G with n vertices: $1 \leq \dim(G) \leq n$-1.
- For a graph with diameter d: $\dim(G) \leq n - d$.
- **Symmetry Impact**: Highly symmetric graphs (e.g., complete graphs, cycles) tend to have higher metric dimensions because symmetries make vertices harder to distinguish[52].

Computational Notes

- **NP-Hardness**: Determining the metric dimension of an arbitrary graph is NP-hard[53].
- **Special Cases**:
  - **Trees:** Can be computed in polynomial time using algorithms based on leaves (pendant vertices).
  - **Cycles:** $\dim(C_n) = 2$ for $n \geq 3$.
  - **Wheel Graphs:** $\dim(W_n) = \lfloor(2n + 2)/5\rfloor$ for $n \geq 7$[54].

Metric Dimension and Its Variants

Several variations of metric dimension have been studied based on different constraints:

Classical Metric Dimension

The *metric dimension* of a graph G is the smallest number of vertices (called *landmarks* or *resolving set*) needed such that every vertex in G is uniquely identified by its distances to these landmarks. Formally, a subset $S \subseteq V(G)$ is a *resolving set* if for any two distinct vertices $u,v \in V(G)$, there exists at least one vertex $s \in S$ such that:

$$d(u,s) \neq d(v,s)$$

where $d(u,s)$ denotes the shortest-path distance between u and s.

The metric dimension of G, denoted $\dim(G)$, is the minimum cardinality of such a resolving set[55].

Key Points:

- **Distinguishing Vertices:**
  - Two vertices u and v are *distinguished* by a landmark s if their distances to s are different.
  - A resolving set must ensure that every pair of distinct vertices is distinguished by at least one landmark[56].
- **Example:**
  - In a path graph P_n, the metric dimension is 1 because one endpoint can distinguish all vertices (each vertex has a unique distance to it)[57].
  - In a complete graph K_n, the metric dimension is $n-1$ because no two vertices can be distinguished unless almost all are landmarks[58].
- **Applications:**
  - Network verification (unique node identification)[59].
  - Robot navigation (landmarks help determine position in a space modeled by a graph)[60].
  - Chemistry (identifying molecular structures)[61].

Local Metric Dimension:

The local metric dimension of a graph G is the smallest number of landmarks (vertices) required such that every pair of adjacent vertices is distinguished by at least one landmark. Formally, a subset $S \subseteq V(G)$ is a *local resolving set* if for any two adjacent vertices $u,v \in V(G)$, there exists at least one $s \in S$ such that:

$$d(u,s) \neq d(v,s),$$

where $d(u,s)$ is the shortest-path distance between u and s.

The local metric dimension of G, denoted $\dim_L(G)$, is the minimum size of such a set S[62].

Key Differences from Classical Metric Dimension:

- **Focus on Adjacent Vertices:**

  - **Classical:** Must distinguish all pairs of distinct vertices[63].
  - **Local:** Only needs to distinguish adjacent (directly connected) vertices[64].
- **Always ≤ Classical Metric Dimension:**
  - Since local resolving sets have a weaker condition, $\dim_L(G) \le \dim(G)$[65].
- **Graphs Where They Are Equal:**
  - In graphs where every pair of vertices is distinguished by adjacency (e.g., complete graphs K_n), the local and classical dimensions coincide[66].

Examples:

1. **Path Graph P_n:**
   - **Classical:** $\dim(P_n)=1$ (one endpoint suffices)[67].
   - **Local:** $\dim_L(P_n)=1$ (same, since adjacent vertices are distinguished by the endpoint)[68].
2. **Cycle Graph C_n:**
   - **Classical:** $\dim(C_n)=2$ (two landmarks needed to break symmetry).
   - **Local:** $\dim_L(C_n)=2$ for $n \ge 4$, but $\dim_L(C_3)=2$ (since $C_3 \equiv K_3$)[69].
3. **Complete Graph K_n:**
   - **Classical:** $\dim(K_n)=n-1$ (need $n-1$ landmarks to distinguish all pairs).
   - **Local:** $\dim_L(K_n)=n-1$ (since every pair is adjacent, same as classical)[70].
4. **Star Graph S_n:**
   - **Classical:** $\dim(S_n)=n-1$ (must mark all but one leaf to distinguish them).
   - **Local:** $\dim_L(S_n)=1$ (just the center distinguishes all adjacent pairs)[71].

Applications:

  - **Network Monitoring:** Ensuring that connected nodes (e.g., routers) can be uniquely identified[72].
  - **Chemical Graph Theory:** Differentiating bonds (adjacent atoms) in molecular structures[73].
  - **Error Detection in Networks:** Identifying faulty connections between adjacent nodes[74].

Strong Metric Dimension

The strong metric dimension of a graph G is the smallest number of landmarks (vertices) required such that every pair of distinct vertices is distinguished by at least one landmark in a "strong" way: either one landmark lies on a shortest path between them, or they have different distances to at least one landmark.

Formally, a subset $S \subseteq V(G)$ is a *strong resolving set* if for any two distinct vertices $u,v \in V(G)$, there exists $s \in S$ such that:

s lies on a shortest u-v path, or $d(u,s) \neq d(v,s)$.

The strong metric dimension of G, denoted $\mathrm{sdim}(G)$, is the minimum size of such a set S [55].

Fault-Tolerant Metric Dimension

Definition:

- A resolving set S is **fault-tolerant** if it remains a resolving set even after the removal of any single landmark.
- The fault-tolerant metric dimension is the smallest such S [75].

Key Properties:

- Requires redundancy: $\dim FT(G) \ge \dim(G)+1$.
- Example:
  - For a path P_n, $\dim FT(P_n)=2$ (both endpoints must be selected).
  - For a cycle C_n, $\dim FT(C_n)=3$.

Applications:

- Robust network monitoring where landmarks may fail[72].

Mixed Metric Dimension

Definition:

- A set S resolves both **vertices and edges**, meaning:
  - Every vertex is uniquely determined by distances to S.
  - Every edge (u,v) is uniquely determined by distances to S.
- The smallest such S is the mixed metric dimension[76].

Key Properties:

- $\dim_M(G) \geq \dim(G)$.
- **Example:**
  - For a path P_n, $\dim M(P_n)=2$.
  - For a cycle C_n, $\dim M(C_n)=3$.

Applications:

- Network design where both nodes and connections need identification[77].

Edge Metric Dimension

Definition:
- A set S resolves edges if every edge (u,v) has a unique distance vector to S.
- The smallest such S is the edge metric dimension, $\dim_E(G)$[78].

Key Properties:
- Different from classical dimension: Some graphs have $\dim_E(G) < \dim(G)$.
- **Example:**
  - For a path P_n, $\dim E(P_n)=1$.
  - For a cycle C_n, $\dim E(C_n)=2$.

Applications:
- Distinguishing connections (e.g., in transport networks)[79].

k-Metric Dimension

Definition:
- A set S is a k-resolving set if every pair of vertices is distinguished by at least k landmarks.
- The smallest such S is the k-metric dimension, $\dim_k(G)$[58].

Key Properties:
- Generalizes classical (k=1) and fault-tolerant (2k=2) cases.
- Example:
  - For a complete graph K_n, $\dim_k(K_n)=n-1$.

Applications:
- Higher reliability in sensor networks[80].

Multiset & Step Metric Dimension

Multiset Version:
- Considers multisets of distances rather than single distances.
- Useful when uniqueness is hard to guarantee[81].

Step Metric Dimension:
- Uses distances up to a fixed radius r.
- Example: In social networks, only local (step-limited) information may be available[82].

Fractional Metric Dimension

Definition:
- Allows "partial" landmark assignments (weights between 0 and 1).
- The fractional dimension is often smaller than the integer version[83].

Applications:
- Optimizing landmark costs in large networks [84].

Outer & Connected Metric Dimension

Outer Version:
- Landmarks must lie in a specified subset (e.g., boundary vertices)[55].

Connected Version:

- The resolving set must induce a connected subgraph [85].

Applications:

- Geographic networks (e.g., border surveillance)[86].

CONCLUSION

The metric dimension has developed from a theoretical construct of graph theory into a cross-disciplinary measure of wide ranging implications. Defining the number of landmarks needed to uniquely identify elements in a network, this paradigm provides neat solutions for many network design, robotics, chemistry, biology, and computer security problems.

Some of the most significant findings revealed by this research are:

- **Theoretical Backgrounds:** The metric dimension varies extensively across graph families—from highly uncomplicated examples like paths to extremely complicated graphs like product graphs—highlighting rich interconnectivity with symmetry, diameter, and other invariants of graphs.
- **Algorithmic Challenges:** It is NP-complete to calculate the metric dimension for general graphs, although effective algorithms exist for special cases (e.g., trees), facilitating further investigation of approximations and heuristics.
- **Practical Impact**: Applications such as sensor placement, genome sequencing, and intrusion detection demonstrate how abstract combinatorial principles translate into real-world optimizations.
- **Emerging Directions**: Variants like edge, fractional, and fault-tolerant metric dimesions address specialized needs, while open problems (e.g., dynamic graphs, higher-dimensional analogs) suggest fertile ground for future research.

As networks grow in size and complexity, the demand for efficient identification and localization strategies will only intensify. The metric dimension offers a unifying lens to tackle these challenges, bridging theory and practice in an increasingly interconnected world. Future work could explore machine learning-aided resolving sets or quantum graph embeddings, further expanding the boundaries of this field.

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